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Earnings and liquidity factors

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A model with factors for earnings, liquidity, their respective growth, and the market can offer a consumption rationale with low pricing error. It also subsumes one-year momentum and momentum net of reversal, the factor commonly known as 'momentum.' These earnings and liquidity factors are all significant and combine for a model without factor redundancy. Motivated by investors' ability to establish positions, we construct portfolios based on volume, and reconcile liquidity into reduced form characteristics-based factor models that compliment firm-based factors.

1. Introduction

Volume and earnings information, along with prices, have dominated financial reporting for over one hundred years. Financial economists have constructed factor models with characteristics related to earnings, liquidity, (to a lesser extent volume) and the market, with price momentum usually viewed as a puzzle. We build three, four and five factor models with earnings, liquidity (volume as proxy), their growth, and the market as return factors. The same characteristics motivating investors' historical information purchases allow for a model that in some cases obtains 'no pricing error' even while partitioning with NYSE breakpoints. These factors can also subsume one-year momentum and momentum net of reversal. Candidate benchmark factor models often do not include liquidity, but we find the inclusion of the simple characteristic of volume allows construction of a reduced form factor model with five factors that performs better in asset pricing tests and against the most common anomalies relative to the current state of the art models.

While Fama and French (2015) is empirical and is not explicitly a 'production based' model they use the firm-based factors of book-to-market, size, profitability and investment. We take a complimentary view of investors concerned about consumption wanting to manage their own earnings and liquidity risk. Our factors for the market, earnings, liquidity, and the latter two's growth rates have received much theoretical motivation in the literature. Snigaroff and Wroblewski (2020) include mixes of several of these variables to form a consumption asset pricing model that is not rejected when tested against well-known consumption 'puzzles.'

We define a return based five-factor model which we refer to as an (ELM) model and build three, four and five factor versions. The incorporation of liquidity (we assume our volume characteristics are liquidity proxies – an assumption that has had mixed reception) integrates the liquidity literature, but also allows the model to subsume one-year momentum, and momentum net of one-month reversal, the Carhart (1997) factor commonly known in the literature as 'momentum.' Liquidity's relation to momentum was a feature of the Pástor and Stambaugh (2003) study, but Li, Novy-Marx, and Velikov (2019) find their measure "essentially unrelated to momentum" (p. 223) as does Pontiff and Singla's (2019) replication study. We demonstrate relatively low pricing errors with respect to time series and cross-sectional pricing error tests. As the ELM model is most like Fama and French's body of work (1993, 1996, 2015), who also model in levels, and as their research is highly influential, we construct our model in the same way to facilitate easiest evaluation[1] .

The combination of *value*, *size* and the *market* as factors (Fama & French, 1993, 1996) has been one of the most influential models

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[1] González and Jareño (2019) do extend Fama & French models with liquidity, albeit with a quantile regression design. Their goal is to extend Fama and French (1993, 2015) models while our aim is to provide differently conceived alternatives.

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in financial economics. While value has been proposed as a risk to consumption (see Parker and Julliard, 2005; Fama and French, 1995, 1996) have generally related the factor premium to "distress." While it is unreasonable to expect a model with a few factors to stave-off anomaly challenges, the *momentum* anomaly has been especially pernicious. Additions to these factors in Fama and French (2015) include *profitability* and *investment*, which they motivate by an internal rate of return argument. Their motivation is disputed by Hou, Mo, Xue, and Zhang (2018) who have made their own four factor model in Hou, Xue, and Zhang (2015) based on a production argument. But neither model subsumes momentum, which Fama and French (2018) add for a sixth factor "somewhat reluctantly" as it "lack[s] theoretical motivation" (p. 237). While Fama and French (1993, 1996, 2015) have emphasized the empirical nature of their models, both they and Hou et al. (2015) model firm level risk.

A close alternative to value is earnings, and for size is liquidity. Here we opt for investor motivated factor proxies, and we revisit the use of *E/P*, and combine with a simple liquidity proxy – volume, for several reasons.

First, we wish to overcome the data-mining critique which Harvey (2017), argues is pervasive to the literature. Investors and market makers have long cared to know market volume information. Informal text searches of *The Wall Street Journal* and *The New York Times'* editions for the latter half of the nineteenth century show daily reporting of stock volume, that continues today. News regarding earnings and earnings changes were reported much more frequently than book value and market capitalization, with the latter always related to initial floatation size.[2] The almost real time reporting of price and volume information was an important revenue source accommodating the rapid growth of communication technology, see for example, Field (1998). Factors need not relate to characteristics, but characteristics that are demonstrably important to investors ought to be exhaustively researched, even if ultimately: we cannot explicitly identify them as 'the true' risk factors, we reject them, or they are not as clean today theoretically. Characteristics that dominate financial reporting for so long, including prior to the study period, are not data mined.

Secondly, while earnings and HML[3] are closely related, earnings are a first order consideration for investors concerned about their own earnings risk and consumption. A model with earnings and liquidity allows us to think about model motivation from the buyer's perspective, rather than the producer's. We can think of investors who care about earnings and earnings and liquidity risk, rather than distressed producers, hence distressed investments to investors. Financial and other crisis are accompanied by shocks to employment and investors want to hedge this risk. Earnings and the ability to hedge earnings risk via a liquid market may be a primary concern of investors. These have a long literature with early examples including Merton (1971) on personal earnings risk, and Tobin (1958) on liquidity risk. We discuss the relation of our factors to firm characteristic factors in Section 2.6. As to dividends or earnings, we rely on Miller and Modigliani (1961), and further, Boudoukh, Michaely, Richardson, and Roberts (2007) and DeLisle, Morscheck, and Nofsinger (2020) who show buybacks and non-dividend payments have become much more prevalent and dividends a less reliable measure in explaining returns than earnings.

Liquidity is different than SMB, and by bringing liquidity into our model we acknowledge the vast theoretical and empirical liquidity literature. Further, changes in liquidity allows the displacement of momentum factors, which Fama and French (2015) or Hou et al. (2015) do not achieve.

For our liquidity measure, we use the simple characteristic of dollar volume. Similar to Amihud (2002), we use a level – the dollar volume of trade to measure liquidity across stocks. While Amihud used a ratio of price change to volume, Lou and Shu (2017) finds the measure is driven by volume. Investors may gauge the intensity of others' views on a security by volume,[4] see Gervais, Kaniel, and Mingelgrin (2001) for a volume-based model. Investors may gauge the health of financial intermediation via volume, the quantity made available in the market produced by the financial sector, as in Lo and Wang (2006).

We also use the change in volume as a dynamic measure of the changing relative risk between assets. To more effectively measure this, monthly rebalancing is useful, and makes the liquidity growth proxy a powerful factor. Li et al. (2019) say Pástor and Stambaugh's (2003) annual rebalancing "seems somewhat strange" (p. 234) and find their model set-up sensitive to rebalancing frequency and argue for better model design with "the natural, monthly frequency" (p. 234). Blume, Easley, and O'hara (1994) argue volume and the magnitude of price changes are theoretically connected, where volume is a crucial part of investor's information set and important in equilibrium modeling. They model high volume from either good or bad news about a stock. An innovation we provide is an effective treatment of high volume related to bad news. Importantly, this allows an alternative way to view the momentum puzzle by modeling investors concerned with assets moving relative to their exposure to the changing state of liquidity.

In Section 3 we show that ELM fits data well with respect to redundancy and asset pricing tests. We then show ELM performs similarly at explaining eight anomalies shown on Kenneth R. French's as Fama and French (2015). In our final section we show the ELM model subsumes momentum factors. Lee and Swaminathan (2000) and Asness, Moskowitz, and Pedersen (2013) demonstrate momentum's relation to liquidity, and Hong and Stein (2007) provide an interesting theoretical relation. These authors, and our findings, are consistent to our model's assumption of compensation for consumption risk: the risk to improvements in liquidity and earnings are important to investors' forward-looking wealth and possible consumption sets and their increases may be largely what price momentum measures.

While we assume rational investors pricing risk, our tests, do not distinguish between a behavioral or rational view of asset prices; e.g., volume and volume changes could measure sentiment. But as we bring liquidity front and center into asset prices our work here is consistent with the financial intermediation view, where the finance sector is especially important. This is compelling, as it is consistent to Muir (2017).

2. An earnings, liquidity, and market model

2.1. The cross section

We construct variables and factors for the Market, Earnings-to-Price, Liquidity, Earnings-Growth-to-Price, and Liquidity Growth. We denote these variables by *MKT, E/P, LIQ, EG/P,* and *LIQG*, respectively. Return difference factors are indicated by a subscript *R* for each variable, e.g., E/P_R.[5] We begin by building three, four and five factor models and describe the fits for each model combination in relation to the Fama and French (2015) five factor model due to its similarity. For the *LIQ* and *LIQG* variables we use dollar volume and weight on dollar volume. A factor model applicable to the broad market must be investible and we could have instead constructed *LIQ* based on turnover and made portfolios investible by

[2] Price and price changes (price momentum) are reported on as well.

[3] *HML* is the value, and *SMB* is the size factor. We refer to Fama and French factors according to their nomenclature.

[4] Similar to internet users gauging efficacy by others' traffic.

[5] E/P_R refers to $(E/P)_R$, EG/P_R refers to $(EG/P)_R$, B/M_R refers to $(B/M)_R$ throughout.

weighting on earnings. But investors' ability to establish and exit trade positions makes dollar volume a more attractive weighting scheme. Alternatively, we could have used the common market capitalization-based weighting scheme, but that does not line up perfectly with ability to establish positions. While we do not show it, we constructed market cap versions of our factor model and while the results were not in general as strong as shown here, the market cap versus liquidity weighting are not the primary driver of our results. We also constructed annually rebalanced volume weighted Fama and French (2015) and Hou et al. (2015) factors, but as those premiums are broadly similar to capitalization weighted factor premiums, and in the interest of space, we do not report them. While we believe it is most correct to volume weight given our factors, it is useful to know that it is primarily the factor selection that is most important in model set-up, not the weighting scheme.

Earnings and liquidity as macro-risks benefit with the additional descriptive power of growth variables in a cross-sectional model. We add rates of change to obtain *EG/P* and *LIQG* to proxy for the risk to increasing earnings and liquidity. Fama and French (2015) also include a related growth factor. Their *investment* factor is a proxy for "the expected growth of book equity" (p. 4) but they also test an explicit growth in book equity variable and opt for the investment factor as it produces larger spreads in average returns. Our *LIQG* return, while directly related to *LIQ*, produces a large average return spread and the factors of EG/P_R and $LIQG_R$ provide additional power beyond the related E/P_R and LIQ_R factors. Our five-factor model is given by the following equation:

$$R_{i,t} - R_{F,t} = \alpha_i + \beta_i \cdot MKT_{R,t} + \phi_i \cdot LIQ_{R,t} + \pi_i(E/P)_{R,t} + \delta_i(EG/P)_{R,t} + \gamma_i \cdot LIQG_{R,t} + \varepsilon_{i,t}. \tag{1}$$

In addition to the market, factors for LIQ_R, E/P_R, EG/P_R and $LIQG_R$ are included. Since earnings and liquidity may be viewed as state risks the earnings and liquidity factors should perform poorly when there are negative consumption and wealth shocks. Cross-sectionally the stocks that have high beta loadings on these risk factors (high sensitivity to earnings and liquidity shocks) earn larger risk premia to compensate for their poor performance in bad states. This is a demand side perspective of the distress risk view of Fama and French (1993, 1996, 2015). Their well-known view has been that *value* and *small* perform poorly as a compensation for distress risk. If the market, earnings, and liquidity are state risks that investors would like to hedge to minimize impact on their own consumption and wealth, investors must be induced to own high earnings and liquidity exposure stocks to offset their higher exposures to this time varying state risk. Our factor model discussion addresses how well (1) does in adhering to this view.

Regarding LIQ_R and $LIQG_R$, there are now many liquidity models, with most researchers using constructed liquidity factors, or factors with liquidity covariance measures. There is an extensive stock liquidity risk literature. Our dollar volume sort is a simple factor design. Stoll (1978); Glosten and Harris (1988); Brennan and Subrahmanyam (1995), and Brennan, Chordia, and Subrahmanyam (1998) use volume as a liquidity measure. Also, some authors propose volume as a sentiment proxy. In the liquidity literature, the level and liquidity covariance are often modeled as separate risks, for example Pástor and Stambaugh (2003) and Sadka (2006) separate in this way. Here, we model investors' concern about level risk and growth risk as their increases may well be what price momentum measures.

Regarding earnings, there is a long literature relating earnings and earnings growth to returns which we do not cite here. Fama and French (2006) provide an extensive study of expected cash flows' relation to *B/M* but do not find improved measures. Also, in industry, financial intermediaries provide earnings estimates to investors and forecast changes in earnings. Regarding theoretical arguments relating earnings to consumption see Merton (1971); Abel (1999), and many others cited in Snigaroff and Wroblewski (2020).

2.2. Cross-sectional model variables and data

Our stock return and accounting data comes from Compustat using the Research Insight interface. The sample period covers the 604 months from February 1968 through May 2018. The Compustat dataset has monthly frequency data beginning in 1962, but we analyzed the data starting in 1968 because many of the twenty-five sorts and test portfolios were very sparsely populated earlier. We use stocks listed on the NYSE, NASDAQ, or the AMEX exchanges as portfolio constituents. For the breakpoints we use only the NYSE stocks. Sorting first on NYSE is standard in cross-sectional literature as this allows the bucket formation to not be dominated by small firms. Fama and French (2012 p. 470) warn about this as well. Variable definitions are shown in the top panel of Table 1. For the earnings in *E/P*, we use trailing twelve-month earnings per share (Diluted) including extraordinary items (Compustat data item *EPSX12*) for the company. To avoid look-ahead bias for slow reporting firms we use a six-month lag to account for the earnings reporting cycle, standard in cross-sectional studies. Hence the most recent possible earnings included for each stock ends in month $t - 6$. Earnings for a given company are constant over their fiscal quarter. Because companies have different fiscal quarters and we sort each month, the ranks change monthly.

We use both positive and negative earnings firms for calculating the earnings-to-price breakpoints and when forming the liquidity-earnings-to-price portfolios. Although price is known daily, Fama and French (1996) match price to its time corresponding lagged book value for their lagged *B/M* variable. We follow, so price is also lagged by six-months as well in *E/P*. Earnings growth to price, *EG/P*, is the one-year earnings per share at $t - 6$ minus the one-year earnings per share at $t - 18$ all divided by the price at $t - 6$. Liquidity, *LIQ*, is computed for month t by multiplying the end of month t share price by the number of shares traded in the month ending at time t. Share price, the volume of stocks traded, and the number of shares outstanding for companies is assumed to be known by investors at the close of each day. This information was reported on by the investment technology of the day, and in daily newspapers for the sample period we study, 1968–2018. Liquidity growth, *LIQG*, for each month t is the trailing one-year percentage change in dollar volume, $LIQG_t := (\$Volume_t - \$Volume_{t-12}) / \$Volume_{t-12}$ when the cumulative return, from time *t-12* to time *t-1* (eleven months), for a stock is greater than or equal to zero, and when the cumulative return, from time *t-12* to time *t-1*, is less than zero $LIQG_t := -1 \times |(\$Volume_t - \$Volume_{t-12}) / \$Volume_{t-12})|$. As explained in Section 3.4 this converts the v-shaped relationship between volume growth and returns to be 'linear,' such that bad news as proxied by price, does not 'contaminate' the purpose of the premium. We also applied, in the case of *LIQG*, a 1% winsorization to the cross-sectional dollar volume to constrain the high or low extremes relating to this ratio.

Capitalization, or market equity, is stock price times the number of shares outstanding. For our model we add the market return $R_M - R_F$ which we refer to as MKT_R. Note that in our model, MKT_R refers to the market factor as calculated with dollar volume weighting, while ${MKT_R}^{CAP}$ refers to the market factor with a capitalization weighting scheme followed by Fama and French (1993) and other researchers. Both include negative-dollar earnings stocks for return calculation. R_F is the one-month Treasury bill rate of return. It would be ideal to use expected earnings and expected liquidity, but even though

Table 1
Cross-Sectional Definitions and Portfolio-Factor Component Construction.

Variable	Description	Calculation
MKT	Market	$R_M - R_F$ *(LIQ weighted)*
E/P	Earnings-to-Price	$\text{One year } EPS_{t\text{-}6} \; / \; Price_{t\text{-}6}$
LIQ	Liquidity	$\text{Monthly shares traded}_t \times \text{Month Price}_t$
EG/P	Earnings-Growth-to-Price	$\dfrac{\text{One year } EPS_{t\text{-}6} - \text{One year } EPS_{t\text{-}18}}{Price_{t\text{-}6}}$
LIQG	Liquidity Growth	$\begin{Bmatrix} \dfrac{LIQ_t - LIQ_{t\text{-}12}}{LIQ_{t\text{-}12}} & \text{if } (Cumret_{t\text{-}12,t\text{-}1} \geq 0) \\ -\left\lvert \dfrac{LIQ_t - LIQ_{t\text{-}12}}{LIQ_{t\text{-}12}} \right\rvert & \text{if } (Cumret_{t\text{-}12,t\text{-}1} < 0) \end{Bmatrix}$

		E/P, EG/P, LIQG			
		Low L 30%	Neutral N 40%	High H 30%	Independent 2 x 3 stock sorts, grouped into Liquid or Illiquid and intersected with L, N, H which refers to Low, Neutral, or High. LIQ weighted portfolios .
LIQ	Illiq 50%				
	Liq 50%				

Exposure	Factor	NYSE Breakpoints	Factor Components
β	MKT_R	None	$R_M - R_F$, *Weighted by LIQ*
π	E/P_R	30th and 70th percentiles	*(IlliqH + LiqH)/2 − (IlliqL + LiqL)/2*
δ	EG/P_R	30th and 70th percentiles	*(IlliqH + LiqH)/2 − (IlliqL + LiqL)/2*
γ	$LIQG_R$	30th and 70th percentiles	*(IlliqH + LiqH)/2 − (IlliqL + LiqL)/2*
ϕ	LIQ_R	Median	$(LIQ_R^{E/P} + LIQ_R^{EG/P} + LIQ_R^{LIQG})/3$
	$LIQ_R^{E/P}$	Median	*(IlliqH + IlliqN + IlliqH)/3 − (LiqH + LiqN + LiqL)/3*
	$LIQ_R^{EG/P}$	Median	"
	LIQ_R^{LIQG}	Median	"

Data is from Compustat. R_M is the market return and R_F is the Treasury Bill return for the month. Returns are calculated monthly. EPS_t is the trailing one-year earnings per share of each stock ending in month *t*. Stocks on all exchanges are *LIQ* weighted based on NYSE breakpoints. Table 1 shows calculation of $LIQ_R^{E/P}$, which is the return difference for the average *Illiq* minus the average *Liq* portfolios when sorted on *E/P*. Similarly, $LIQ_R^{EG/P}$, and LIQ_R^{LIQG} are return differences for average *Illiq* minus average *Liq* portfolios when sorted on *EG/P* or *LIQG*. The general LIQ_R factor is the average of these three.

the finance industry forecasts earnings, its use would dramatically lower the size of our universe.

2.3. Average returns

Before we construct a linear factor model, we first look at average excess returns for portfolios. Ideally, the variables we use to produce factors can be used to sort portfolios to demonstrate large return spreads over the 604 months of data. The universe of stocks is assembled into twenty-five portfolios in several different ways and their mean returns and *t*-stats are shown in Table 2, Panels A–C. Panel A forms the twenty-five portfolios by intersecting five liquidity quintiles, *LIQ*, and five *E/P* quintiles. Panel B intersects the sorts by *LIQ* and *EG/P*, and Panel C intersects by *LIQ* and *LIQG*. As the sort mechanics are the same, our Table 2 is comparable to Table 1 of Fama and French (2015, p.3), although time spans and universes are inexact. As Fama and French (2015)) references are used frequently in the following discussion, we refer to that particular study as "FF." In Panels A–C, the average spread between most Illiquid and most Liquid (the average of the top – the bottom row) is a monthly percent return of 0.48, 0.48, and 0.60 for *E/P*, *EG/P* and *LIQG*, respectively. These are materially higher than the FF spreads between Small and Big (the average of the top – the bottom row in their Table 1 Panels A–C) of 0.31, 0.41, and 0.32 for *B/M*, *OP*, and *INV*, respectively (book-to-market, profitability, and investment). The *LIQ* sorts on dollar-volume work quite well relative to size. For the spreads along the other dimensions, the averages of the high – low columns (right – left) for *E/P*, *EG/P*, and *LIQG* are 0.37, 0.19, and 0.73 percent per month. FF have 0.48, 0.31, and -0.41 for *B/M*, *OP*, and *INV*, respectively. The absolute value of the high – low columns for the two sorting schemes are almost the same on average. But there are some interesting individual differences.

FF find "a glaring outlier" (p. 3) in the micro-cap column of their Size-B/M portfolios, "lethal" (p. 4) low average returns for micro with high investment, and in general, "Because the characteristics are correlated, the Size-B/M, Size-OP, and Size-Inv portfolios in Table 1 [their average return table] do not isolate value, profitability, and investment effects in average returns (p. 4)." They show a particularly high correlation of 0.70 between their investment factor, *CMA*, and *HML*. Our factor correlations are shown in Table 3, and the highest correlation between non-market factors is 0.40. This helps our average return matrix in Table 2 to show well behaved spreads that are consistent with our five-factor model. In Panels A, B, and C, every column of *Illiquid* minus *Liquid* produces a meaningfully positive return spread, and every row of *High* minus *Low* for *E/P*, *EG/P*, and *LIQG* produces positive return spreads.

We also do not suffer from any 'glaring outliers' in the way the analogous FF table does. That is, in each of our panels A–C, we do not have any portfolio return in the most Illiquid column for which it has a lower return in its corresponding most liquid column. And we do not see any return in a particular row in the farthest right *High* column having a lower return than its corresponding leftmost *Low* row. FF's Table 1 has two such glaring outliers where the smallest quintile in a column has lower returns than the largest. Neither ours nor their sets of 75 portfolios have high – low 'glaring' outliers.

Return spreads for our neighboring portfolios are not always monotonic. If we classify as a 'moderate outlier' any neighboring *LIQ* average return spreads (values directly below) as being a double-

Table 2
Excess Returns on 25 Stock Portfolios. 1968-2018, 604 months.

	Low	2	3	4	High	Low	2	3	4	High
			Arithmetic Mean (%)					T-Stat		
Panel A: Liquidity-E/P Sorts										
Illiquid	**1.02**	**0.90**	**0.95**	**1.07**	**1.35**	**3.01**	**3.60**	**4.25**	**5.31**	**6.10**
2	**0.74**	**0.84**	**0.84**	**0.90**	**1.19**	**2.23**	**3.23**	**3.89**	**4.24**	**5.30**
3	**0.75**	**0.84**	**0.81**	**0.95**	**1.09**	**2.26**	**3.50**	**3.86**	**4.70**	**4.76**
4	0.60	**0.75**	**0.73**	**0.83**	**0.92**	1.85	**3.18**	**3.51**	**4.16**	**4.14**
Liquid	0.36	**0.64**	**0.55**	**0.58**	**0.77**	1.17	**2.85**	**2.73**	**2.99**	**3.44**
Panel B: Liquidity-EG/P Sorts										
Illiquid	**1.12**	**0.70**	**0.90**	**0.96**	**1.32**	**3.77**	**3.14**	**4.40**	**4.39**	**4.84**
2	**0.86**	**0.82**	**0.80**	**0.95**	**0.98**	**2.96**	**3.66**	**3.72**	**4.17**	**3.60**
3	**0.85**	**0.73**	**0.82**	**0.96**	**1.01**	**2.91**	**3.35**	**3.92**	**4.20**	**3.64**
4	**0.76**	**0.66**	**0.74**	**0.83**	**0.86**	**2.73**	**2.99**	**3.56**	**3.63**	**3.20**
Liquid	0.27	0.38	**0.63**	**0.68**	**0.65**	1.00	1.60	**3.00**	**2.87**	**2.44**
Panel C: Liquidity-LIQG Sorts										
Illiquid	**0.71**	**0.83**	**0.87**	**1.10**	**1.54**	**2.59**	**3.81**	**4.34**	**5.38**	**6.65**
2	**0.65**	**0.60**	**0.76**	**0.89**	**1.26**	**2.25**	**2.58**	**3.60**	**4.20**	**5.24**
3	0.52	**0.64**	**0.80**	**0.91**	**1.23**	1.86	**2.65**	**3.63**	**4.24**	**4.90**
4	0.32	**0.55**	**0.69**	**0.83**	**1.01**	1.19	**2.33**	**3.06**	**3.86**	**3.94**
Liquid	−0.06	0.35	**0.50**	**0.54**	**0.73**	−0.25	1.48	**2.35**	**2.51**	**2.79**

We construct twenty-five portfolios at end of each month. Returns are calculated monthly and are then averaged across the 604 months. For the portfolio construction of *E/P*, *EG/P*, and *LIQG* and their Liquidity cross sorts see Table 1. For Panels A–C stocks are *LIQ* weighted and the quintiles are formed based on NYSE *LIQ* breakpoints. Means with significant t-statistics (with 95% confidence) are in bold.

Table 3
Factor return summary 1968-2018.

Panel A: Summary Statistics of the Five Factors.	MKT_R	LIQ_R	E/P_R	EG/P_R	$LIQG_R$
Arithmetic Mean (%)	0.55	0.32	0.30	0.26	0.58
Standard Deviation (%)	5.56	2.76	3.76	2.31	3.26
t-statistic	*2.41*	*2.88*	*1.97*	*2.74*	*4.34*

Panel B: Correlation Matrix for the Five Factors.	MKT_R	LIQ_R	E/P_R	EG/P_R	$LIQG_R$
MKT_R	1.00	−0.17	−0.59	0.02	−0.10
LIQ_R	−0.17	1.00	0.10	−0.10	−0.15
E/P_R	−0.59	0.10	1.00	0.22	0.13
EG/P_R	0.02	−0.10	0.22	1.00	0.40
$LIQG_R$	−0.10	−0.15	0.13	0.40	1.00

Panel C: Correlation Matrix for the LIQ_R Factors.	$LIQ_R^{E/P}$	$LIQ_R^{EG/P}$	LIQ_R^{LIQG}	LIQ_R
$LIQ_R^{E/P}$	1.00	0.93	0.91	0.97
$LIQ_R^{EG/P}$	0.93	1.00	0.91	0.97
LIQ_R^{LIQG}	0.91	0.91	1.00	0.96
LIQ_R	0.97	0.97	0.96	1.00

Panel A reports descriptive statistics for the five factors and Panel B reports their correlations. *E/P*, *EG/P*, and *LIQG* each have a corresponding *LIQ* factor. In Table 1 we show the calculation of $LIQ_R^{E/P}$, which is the return difference for the *Illiq* minus the *Liq* portfolios when sorted on *E/P*. Similarly, $LIQ_R^{EG/P}$, and LIQ_R^{LIQG} are the return differences for *Illiq* minus *Liq* portfolios when sorted on *EG/P* or *LIQG*. The general LIQ_R factor is the average of these three. The correlations of the individual LIQ_R factors are given in Panel C.

digit sign in the wrong direction, we have one exception (where the return for less *LIQ* is lower by 0.10 or more than the more *LIQ* directly below it) whereas their neighboring *Size* average return spreads produce four moderate outlier double digit exceptions.

Our neighboring *E/P*, *EG/P*, and *LIQG* average return spreads produce four double digit exceptions (where the return for *High-'er'* is below its *Low-'er'* neighbor directly to its left) whereas FF's neighboring return spreads produce no double digit exceptions. These four high – low moderate outliers (e.g., the 1.12 in Panel B) have interesting economic interpretation; they are in the far-left column for *E/P* and *EG/P* sorts. That is, they are portfolios consisting of firms that have the lowest earnings or earnings growth. They are firms that might not survive. The portfolios in far left of the *EG/P* sort of Panel B have higher observed returns than their neighbors, more so than in the other sorts. In all but the most liquid quintile, there is a non-linear return pattern moving across *EG/P*, where the lowest *EG/P* quintile stocks realized higher returns. These patterns relate to Campbell, Hilscher, and Szilagyi (2008), and Gao, Parsons, and Shen (2017) who find lower returns for distressed firms. It may be interesting to study distress' relation to low earnings growth. Our factor model motivation and construction may miss some default, firm quality or distress risk. In any case, unconditional earnings ties directly to our motivation and we limited ourselves to five factors. In Panel C, when the intersections are on *LIQ* with *LIQG*, we again see a largely monotonic increase from left to right.

The average difference of the *EG/P* sorts between the high – low is 0.19 % per month. This is the lowest average difference in any of our three sorts. Earnings growth relates to the investment factor of FF. A firm's investment is the growth of total assets, a proxy for expected growth of book equity. FF replicated, but do not report, the average returns with the growth of book equity and obtained similar results to their published model. All else equal, a high growth of investments or book equity means the firm retention rate is higher, and investment has been proposed as a production variable in Hou, Xue and Zhang's (2015) factor model. It is not as clear from investors' perspective that firm's investment is a risk to consumption. Hence, we use the growth of earnings, *EG/P*, as it is clearer as a consumption risk factor. As expected, the average returns of these two related variables run in opposite directions. Also, Snigaroff and Wroblewski (2020) find both earnings growth and liquidity growth variables allow a consumption-based asset pricing model to not be rejected, unlike a model with only the market.

Our *E/P* sorts have roughly similar *E/P* premiums for all liquidity quintiles, while *EG/P* has higher spreads for the more liquid portfolios and smaller spreads in middle quintiles, due again to the relatively high left column returns just discussed. The *LIQG*, Panel C, also has roughly similar *LIQG* premiums for all liquidity quintiles.

The pattern for Liquidity growth, *LIQG* in Panel C shows consistently increasing returns as we move across the right for higher *LIQG* portfolios. This is consistent with *LIQG* being a risk factor. A behavioral story would suggest underappreciated or unnoticed stocks, low *LIQG*, with higher returns: or overtrading in exuberant names leading to low returns for high *LIQG*. These kinds of patterns do not manifest, and Panel C is consistent with rational pricing. This is encouraging for a view of *LIQG* as a risk factor with investors concerned about the risk of liquidity growth. As our liquidity growth factor is closely related to momentum, this gives some indication of liquidity growth as a possible risk premium.

In summary, our various *LIQ* sorts work well; they make wider return spreads than size. Our characteristic sorts of high – low for *E/P*, *EG/P*, and *LIQG* produce quite similar magnitude spreads as Fama and French's (2015) characteristics that are better behaved. This is a competitive comparison of average returns. It is also an intuitive way to describe stocks. Academic researchers describe *growth* stocks as stocks that have low "*HML*" (value) exposure, whereas practitioners think of *growth* as stocks with high earnings growth (or price momentum). In our classification, growth stocks are those with the characteristic of high earnings or liquidity growth, also with high factor exposures.

2.4. Five-factor model

In this Section we build a five-factor model based on *E/P*, *EG/P*, *LIQ*, *LIQG*, and *MKT*. We reform portfolios monthly. Snigaroff and Wroblewski (2018) build a five-factor model which uses four variables similar to our variables and a fifth variable related to liquidity growth which is different than our variable. Their paper uses annual rebalancing, and their three-factor version provides an interesting comparison to Fama and French's (1993, 1996) three factor model, when the same universe and dates are applied. We find that monthly rebalancing is required to effectively isolate the effect of *LIQG* and *EG/P*. This model also subsumes momentum factors, which Snigaroff and Wroblewski's (2018) model cannot. Factor definitions are shown at the bottom of Table 1. Our factor construction technique replicates Fama and French (1993, 1996, 2015) except: 1) we include the negative earnings firms[6] for breakpoint formation and include them in the portfolios, 2) we rebuild portfolios monthly instead of annually, and 3) as discussed previously – we weight on dollar volume.

We sort stocks independently each year based on *LIQ* and on either *E/P*, *EG/P*, or *LIQG*. Each month all NYSE stocks on Compustat are ranked on *LIQ*. We use the median NYSE *LIQ* to split NYSE, Amex and NASDAQ stocks into two halves: illiquid and liquid, labeled Illiq and Liq. We also break NYSE, Amex, and NASDAQ stocks into either of three groups based on three breakpoints. Depending on their NYSE *E/P*, *EG/P*, or *LIQG* values the bottom 30 % are 'Low,' the middle 40 % are 'Neutral,' and the top 30 % are 'High'. For each factor model we obtain these six sets of stocks: Illiq-Low, Illiq-Neutral, Illiq-High, Liq-Low, Liq-Mid, Liq-High) from the intersections of the two *LIQ* and the three on the other dimension: *E/P*, *EG/P* or *LIQG*. The six portfolios are then separately weighted according to each constituent stock's dollar volume. Each of these three variables has its own corresponding *LIQ* factor, since there are different intersections involved with Ill and Liq for each of the other dimension variables. Table 1 shows the calculation of $LIQ_R^{E/P}$, which is the return difference for the average over Illiq portfolios minus the average over the Liq portfolios when sorted on *E/P*. $LIQ_R^{EG/P}$, and LIQ_R^{LIQG} are the return differences for the average over the Illiq portfolios minus the average over the Liq portfolios when sorted on *EG/P* or *LIQG*. The general LIQ_R factor we use is the average of these three. Table 3, Panel C shows their high correlations.

Monthly returns are calculated for each of the six portfolios and then the portfolios are formed again in the following month. For the *E/P* variable at time *t* we use the earnings over the time-period of month *t – 6* to month *t-18* (to avoid look ahead bias for earnings data). We assign a subscript *R* to differentiate the factor return from the characteristic. LIQ_R (Illiquid minus Liquid), is our risk factor in returns related to liquidity, and is the difference between the returns on illiquid-and-liquid stock portfolios. The portfolio return E/P_R (high earnings to price minus low earnings to price) is the risk factor in returns related to *E/P* and is the difference, each month, between the arithmetic mean of the returns on the two high *E/P* portfolios (Illiq-High and Liq-High) and the arithmetic mean of the returns on the two low *E/P* portfolios (Illiq-Low and Liq-Low). The two components of E/P_R are returns on High and Low *E/P* portfolios with about the same weighted-average liquidity. The same is the case for other factors, EG/P_R and $LIQG_R$. The 2 × 3 sort design which we use gives regression slopes that well isolate the related premiums.

We summarize the five factors in Table 3. The mean for each of the non-market factors range from 0.26 to 0.58 and average 0.36. These are materially higher than for FF's non-market factor means which average 0.28 when measured over the same period. Every factor has a t-stat indicating statistical significance. An important difference with that of the Snigaroff and Wroblewski (2018) annual portfolio formation study is that although they used similar factors with our study here, $LIQG_R$ is now economically and statistically material. Their study shows $LIQG_R$ with a slightly negative mean and insignificant t-stat. Annual rebalancing also gives a fairly flat average return for $LIQG_R$ and the negative factor premium implies a different interpretation for the factor model. We believe monthly rebalancing is a materially better model design to capture the dynamic risk of changes in liquidity. Note that only one of the factors has a *t*-stat greater than 3, as Harvey, Liu, and Zhu (2016) suggest for newly discovered factors. But our only 'newly discovered' factor, $LIQG_R$, has a 4.34 *t*-stat, and all of these factors are motivated from first principals. Both LIQ_R and $LIQG_R$ have high means, but intuitively, one would expect them to be high as compared to corporate bonds, where De Jong and Driessen (2012) find annual liquidity premiums of 0.6%–1.5% (one of their liquidity measures is volume based).

To conduct factor model tests, we construct twenty-five portfolios based on the intersections arising from one of the two univariate quintile sorts. For *E/P* as an example, we form quintiles based on the NYSE breakpoints for *E/P* and then separately form quintiles based on the NYSE breakpoints for *LIQ* and then construct twenty-five intersection (5 × 5) portfolios. We use matrix notation to reference a portfolio. For example, Portfolio (5,5) in the *E/P* set

[6] We suspect Fama and French (1993, 1996, 2015) began their practice of not counting negative *B/M* firms as conservatism in modeling, and they wanted to exclude obviously distressed firms. Negative *B/M* was rarer in earlier periods than now. Snigaroff and Wroblewski's (2018) annual rebalanced model also excluded negative earnings firms in the same way that Fama and French (1993, 1996, 2015) had excluded negative *B/M* firms.

Table 4
LIQ and E/P: regressions of 25 portfolios on *MKT, LIQ, E/P, EG/P, LIQG*.

Five Factors, 1968–2018, 604 months.

$$R_{i,t}\text{-}R_{F,t} = \alpha_i + \beta_i MKT_{R,t} + \phi_i\, LIQ_{R,t} + \pi_i E/P_{R,t} + \delta_i EG/P_{R,t} + \gamma_i LIQG_{R,t} + \varepsilon_{i,t}$$

Liquidity	Panel A. *E/P* Quintiles					Panel B. *EG/P* Quintiles					Panel C. *LIQG* Quintiles				
Quintiles	Low	2	3	4	High	Low	2	3	4	High	Low	2	3	4	High
↓			α					α					α		
Illiquid	0.19	0.03	0.08	**0.20**	**0.39**	**0.30**	−0.09	0.11	0.10	**0.45**	0.05	0.06	0.08	**0.23**	**0.46**
2	−0.09	−0.06	−0.07	−0.04	**0.16**	0.04	0.07	−0.05	−0.01	0.02	−0.01	**−0.22**	−0.10	−0.09	0.10
3	−0.04	0.00	−0.07	0.05	0.07	0.04	−0.01	0.01	0.05	0.00	−0.02	−0.14	−0.06	−0.06	0.05
4	−0.07	−0.01	−0.07	0.02	0.04	0.03	−0.01	0.04	0.02	−0.07	−0.11	−0.06	−0.04	−0.04	−0.09
Liquid	0.05	**0.17**	−0.05	−0.11	−0.05	**−0.20**	0.06	**0.21**	0.10	**−0.19**	**−0.24**	0.10	0.13	−0.12	**−0.21**
			β					β					β		
Illiquid	**1.03**	**0.91**	**0.90**	**0.85**	**0.94**	**1.03**	**0.87**	**0.78**	**0.86**	**0.96**	**0.96**	**0.84**	**0.75**	**0.78**	**0.83**
2	**1.12**	**1.03**	**0.96**	**0.98**	**1.04**	**1.12**	**0.94**	**0.92**	**0.99**	**1.08**	**1.08**	**0.98**	**0.92**	**0.94**	**0.98**
3	**1.19**	**1.02**	**0.97**	**0.98**	**1.10**	**1.17**	**0.97**	**0.95**	**1.02**	**1.14**	**1.07**	**1.06**	**1.02**	**1.01**	**1.05**
4	**1.16**	**1.03**	**0.98**	**0.99**	**1.09**	**1.16**	**1.00**	**0.95**	**1.02**	**1.15**	**1.06**	**1.06**	**1.05**	**1.02**	**1.07**
Liquid	**1.07**	**0.92**	**0.93**	**0.97**	**1.10**	**1.12**	**0.96**	**0.87**	**0.97**	**1.13**	**1.00**	**0.97**	**0.93**	**0.98**	**1.09**
			ϕ					ϕ					ϕ		
Illiquid	**1.65**	**1.26**	**1.13**	**1.04**	**1.14**	**1.46**	**1.06**	**1.02**	**1.06**	**1.33**	**1.28**	**1.09**	**1.02**	**1.03**	**1.16**
2	**1.44**	**1.17**	**0.98**	**0.96**	**1.06**	**1.27**	**0.99**	**0.96**	**1.03**	**1.17**	**1.20**	**1.03**	**0.96**	**0.97**	**1.03**
3	**1.05**	**0.84**	**0.78**	**0.76**	**0.90**	**1.09**	**0.75**	**0.72**	**0.77**	**1.00**	**0.84**	**0.87**	**0.81**	**0.80**	**0.92**
4	**0.60**	**0.51**	**0.53**	**0.54**	**0.59**	**0.73**	**0.47**	**0.43**	**0.50**	**0.64**	**0.45**	**0.54**	**0.54**	**0.52**	**0.64**
Liquid	**−0.23**	**−0.07**	0.03	**0.12**	**0.15**	**0.08**	**−0.18**	**−0.17**	**−0.11**	**0.10**	−0.05	**−0.07**	**−0.08**	0.04	**0.11**
			π					π					π		
Illiquid	**−0.43**	0.00	**0.23**	**0.32**	**0.34**	**−0.08**	**0.13**	**0.20**	**0.17**	**−0.16**	**−0.07**	**0.14**	**0.12**	**0.13**	−0.05
2	**−0.46**	0.05	**0.33**	**0.48**	**0.48**	0.00	**0.18**	**0.24**	**0.27**	−0.03	−0.03	**0.24**	**0.27**	**0.30**	0.01
3	**−0.48**	**0.10**	**0.36**	**0.50**	**0.56**	0.02	**0.22**	**0.26**	**0.23**	0.00	−0.02	**0.29**	**0.34**	**0.38**	0.01
4	**−0.49**	**0.12**	**0.40**	**0.56**	**0.63**	**0.07**	**0.16**	**0.26**	**0.20**	**0.07**	0.03	**0.32**	**0.40**	**0.36**	−0.02
Liquid	**−0.50**	−0.02	**0.34**	**0.56**	**0.71**	**0.08**	−0.03	−0.02	−0.03	**0.10**	0.05	**0.16**	**0.29**	**0.25**	−0.04
			δ					δ					δ		
Illiquid	−0.06	0.00	**−0.09**	−0.01	−0.05	**−0.31**	−0.03	**0.09**	**0.18**	**0.12**	−0.01	−0.07	−0.03	0.00	**0.13**
2	0.06	0.07	−0.04	**−0.11**	−0.02	**−0.34**	**−0.20**	0.02	**0.14**	**0.30**	−0.01	0.01	−0.03	−0.01	**0.09**
3	0.00	**0.11**	**0.07**	−0.04	0.02	**−0.41**	**−0.18**	**0.10**	**0.22**	**0.44**	**0.09**	**0.12**	0.04	−0.04	**0.10**
4	0.00	**0.15**	0.01	**−0.07**	−0.03	**−0.45**	**−0.20**	0.04	**0.30**	**0.32**	**0.15**	0.06	−0.06	−0.01	0.07
Liquid	**−0.21**	**0.07**	0.06	0.02	**0.14**	**−0.91**	**−0.39**	0.01	**0.43**	**0.66**	0.02	0.05	**0.09**	−0.01	**0.11**
			γ					γ					γ		
Illiquid	**−0.19**	**−0.07**	−0.04	−0.04	−0.02	**−0.20**	**−0.10**	**−0.09**	**−0.09**	**−0.10**	**−0.43**	**−0.12**	0.02	**0.14**	**0.40**
2	**−0.21**	**−0.12**	−0.03	**−0.05**	−0.02	**−0.21**	**−0.13**	**−0.06**	**−0.06**	**−0.12**	**−0.52**	**−0.23**	−0.04	**0.11**	**0.46**
3	**−0.09**	**−0.07**	−0.04	−0.03	**−0.08**	**−0.13**	**−0.09**	**−0.07**	−0.04	**−0.08**	**−0.58**	**−0.34**	**−0.11**	**0.10**	**0.50**
4	−0.02	**−0.07**	**−0.06**	**−0.09**	**−0.15**	**−0.09**	−0.04	**−0.08**	**−0.09**	−0.01	**−0.59**	**−0.44**	**−0.22**	**0.06**	**0.52**
Liquid	0.02	−0.03	**−0.07**	**−0.09**	**−0.14**	**0.07**	**−0.05**	0.01	−0.03	−0.01	**−0.64**	**−0.56**	**−0.38**	**0.07**	**0.51**
			R^2					R^2					R^2		
Illiquid	0.85	0.86	0.86	0.88	0.90	0.88	0.86	0.81	0.86	0.89	0.91	0.87	0.82	0.80	0.84
2	0.93	0.90	0.90	0.92	0.93	0.94	0.92	0.89	0.92	0.93	0.95	0.93	0.90	0.90	0.92
3	0.94	0.90	0.90	0.91	0.93	0.92	0.91	0.90	0.91	0.92	0.93	0.92	0.91	0.90	0.92
4	0.92	0.88	0.87	0.88	0.89	0.89	0.92	0.87	0.90	0.89	0.90	0.91	0.88	0.88	0.90
Liquid	0.94	0.90	0.86	0.88	0.86	0.89	0.93	0.93	0.93	0.90	0.88	0.91	0.87	0.87	0.93

Table 4 Regressions statistics associated with twenty-five portfolios regressions on the ELM model. These tables display the five-factor regression coefficients for each of the twenty-five test portfolio regressions. The right-hand side factors are given by *MKT, LIQ, E/P, EG/P, LIQG* in each of the tables (Table 1 displays a detailed construction of these factors). The left-hand side variables are given by the excess return series of each of the twenty-five test portfolios corresponding to a partition of interest. Panel A forms the partitions by intersecting the *E/P* and *LIQ* quintiles. Panel B partitions by intersecting the *EG/P* quintiles with the *LIQ* quintiles. Panel C intersects the *LIQG* quintiles with the *LIQ* quintiles. The columns are labeled from Low to High and indicate the quintiles associated with *E/P, EG/P*, or *LIQG*, while the rows are labeled from Illiquid to Liquid and are based on the *LIQ* quintiles. When the associated t-statistic (not shown) indicates significance (with 95 % confidence) for the intercepts under α the value for the intercept is **bold**, which indicates a problem. When the associated t-statistic indicates significance for the values for β, ϕ, π, δ and γ coefficients are also **bold**, but does not indicate a problem. The sample period is from 1968–2018, 604 months.

of portfolios are stocks from the quintile with the highest liquidity that are also stocks from the quintile with the highest *E/P*. It is a portfolio designed to mimic high earnings and high liquidity exposure. Table 4 displays various results of time series returns regressions in which each of the twenty-five portfolio's returns are regressed on the right-hand-side return factors corresponding to this five-factor model. Rows are labeled by corresponding *LIQ* quintiles (Illiquid down to Liquid) and columns correspond to the quintiles of the second variable sorted on (Low and rightward to High): *E/P, EG/P,* and *LIQG* for Panels A, B, and C respectively. Each portfolio's regression coefficients are labeled by: α, β, ϕ, π, δ, and γ. Their corresponding *t*-stats are not shown. However, when the associated *t*-statistic for α is significant at the 95 % level the value of the intercept is indicated by bold. (Ideally, we want fewer in bold.) When the associated *t*-statistic for the coefficients β, ϕ, π, δ and γ are significant at the 95 % level we also indicate by bold (in this case not a problem).

2.5. Five factor model results

To put our discussion for our five-factor model results shown in Table 4 into context, we relate what we expect to see given our motivation. As per Fama and French (2015) and as is standard in this literature, we form test portfolios as individual stocks are highly noisy. Firstly, the matrices of α's can be thought of as a measure of the five factors' inability to explain the return for the twenty-five test portfolios. Ideally, they are zero or insignificantly different from zero. Since many of the spanning tests utilize these intercepts, we prefer these to be statistically small. Secondly, we posit earnings, growth of earnings, and liquidity are risks to consumption. As E/P_R, EG/P_R and $LIQG_R$ are factors designed to mimic this risk, high *average returns* of Table 2 should reside on the high, or right side of Panels A, B & C. Thirdly, if the factor model well describes portfolio returns it should be the case that when one fixes a particular regression coefficient, π, δ, or γ, corresponding to one of these three risk-factors and then sorts the twenty-five bins by this particular regression coefficient the resulting rankings should positively correlate to high average returns relative to the other bins. While individual multivariate slopes in Panels A–C of Table 4 need not necessarily line up with the average returns in Panels A–C in Table 2, in our case they do. All this generally describes what we observe in Table 4. We now call attention to interesting or problematic exceptions.

The 'growth effect' is that stocks (with low *B/M*) have low abnormal returns. In their five-factor model under the *B/M* sort Fama and French (2015) still show statistically significant alphas for 4 of their 5 left column portfolios (their Table 7)[7] . Their microcap growth stocks have negative returns and their large growth stocks have abnormally positive returns. In Table 4 Panel A, our far-left column has no intercepts that are significantly negative or positive. Our factor model, with controls for *EG/P* and *LIQG* eliminates this 'growth effect' of too low a return for growth stocks. We do not show it, but a three-factor model with MKT_R, E/P_R and LIQ_R (without controls) does show negative intercepts on the left side, similar to Fama and French (2015). This is consistent to our view that the growth of earnings and liquidity are risk proxies.

Unfortunately, the intercepts under α for the most illiquid in the upper right corners of all our sorts are much too high. For our *E/P* sort in Panel A the intercept has a monthly excess, α of 0.39 % with a *t*-stat of 5.18 (and there are three large alphas bolded in the upper right corner), in Panel B for the *EG/P* sort α is 0.45 % with t-stat of 4.61, and in Panel C for the *LIQG* sort α is 0.46 % with a t-stat of 4.68 (with two large alphas in upper right corner). The risk is where it should be, but we have too much of a good thing. The returns for the most illiquid for each sort are too high for our linear factor model to describe. These premiums' existence in the most illiquid names is consistent with the 'investor mistakes' view of Lakonishok, Shleifer, and Vishny (1994) where the difficulty of arbitrage allows them to lurk. In our *EG/P* sort of Panel B, there are intercept outliers in 3 of the 4 corners. The −0.19 % intercept for the most liquid high earnings growth firms is too low. Firms with highest earnings growth with highest trading volume experiencing low returns, fits the popular definition of exuberance. And the 0.30 % intercept for the most illiquid and lowest earnings growth is consistent with overlooked and underpriced firms generating a bias premium.

Our *LIQG* sort in Panel C has several large intercept outliers that are unusual. It is tempting to invent stories around outliers, but we'll forgo doing so on these and the several other outliers. Any of these outliers can be an inference error, but the magnitude of some of these errors causes formal model rejection in most cases – a standard occurrence in cross-sectional models.

Fama and French (1993) show the premium to market return to be homogenous after the addition of *HML* and *SMB*; Beta is flat across test portfolios. Still, leading benchmark factor models require the MKT_R factor to describe returns, as does our model. If we look at the matrix under β in each Panel, we can see that β isn't flat across the twenty-five portfolios but rather it has a U shape. In the *E/P* sort of Table 4 Panel A, the column average for the left and right are 116 % and 109 % of the average over the middle three, in the EG/P sort of Panel B they average 120 % and 116 % of the middle three, and in the LIQG sort of Panel C, 108 % and 105 % of the middle three columns' average. These 'wings' have higher risk that the market factor fails to capture. Fama and French (2015), and our model here, makes these wings more pronounced, and both models say the MKT_R needs to be supplemented. In our case, the negative loadings on the sort characteristic of the far-left columns for all our sorts are material and line up with the high betas on the left in the β matrix. Similarly, the materially positive loadings on the sort characteristics on the right, line up with the high betas on the right in the β matrix.

2.6. Liquidity and earnings, vs. size and value

We follow Fama and French's (1993, 1996, 2015) 2 × 3 framework for several reasons: most direct comparability, factor definitions are not from extreme partitions, and Fama and French (2015) themselves try other kinds of partitions and do not find a material difference. We also follow Fama and French model testing techniques. They have made seminal contributions in model design and testing. Fama and French (1993, 1996, 2015, 2018) take an empirical or ad hoc approach, and their recent research has included accounting level motivation, e.g., see Eqs. 1–3 in Fama and French (2015). Conceptually, two of our factors, LIQ_R and E/P_R, are related to Fama and French's (1993, 1996) *SMB* and *HML*. We look at specific quantitative comparisons of the two models' factors relative to each other throughout Section 3.

Liquidity and earnings are 'flow' characteristics measured over time, whereas *SMB* and *HML* are point in time, or 'stock' characteristics. While our model takes a complimentary investor motivation view to their firm level motivation, this section discusses these two similar factor pairs.

In our model set-up, dollar volume is the weighting scheme and liquidity is front and center. Other researchers' weight by size. Their relation is trivially true, small companies are less liquid. Their relation has also been repeatedly researched since Banz (1981) inaugurated the inception of the size premium literature, e.g., Roll (1981) offered liquidity to explain the small firm effect. However, they are not the same. LIQ_R has 0.69 correlation coefficient with SMB_R[8] over our study period. The similarity between LIQ_R with SMB_R motivates Hou, Xue, and Zhang (2017) argument for widespread *p*-hacking in the liquidity literature; "95 out of 102 liquidity variables (93 %) are insignificant at the conventional 5% level" (p.1). But it is easy to see why researchers may find liquidity insignificant when size is a factor.

Snigaroff and Wroblewski (2018) construct a LIQ_R orthogonalized-to-size factor to show liquidity has grown in importance since 1998. Hazarding a comment about the state of research: rational expectations assumptions of complete information and frictionless markets and representative agent models pushed liquidity arguments into the periphery, and the size factor literature percolated into relative dominance.

However, institutional investors and market index construction suppliers regularly employ 'free-float' weighting schemes. That is, they adjust market capitalization weighting by the number of shares that are not closely held, a revealed preference towards 'tradability.'

A significant problem with size is much, or all, of the premium comes in January, e.g., Lakonishok and Shapiro (1986). This still is the case today. Fig. 1 shows the month by month average premium for SMB_R and LIQ_R, along with the turnover for the NYSE over the period 1969 – 2017. The January average factor return for SMB_R is 1.62 % and for LIQ_R is 2.27 % per month. The average returns for all the non-January months for the SMB_R and LIQ_R factors are 0.02 % and 0.14 % per month, respectively. Ritter's (1988) tax loss selling of mostly small-size losers by individual investors and reinvesting in January is not a size risk argument. His "parking-the-proceeds" conjecture over the turn of the year is a liquidity risk phenomenon. Hong and Yu (2009) find both lower volume and lower returns in fifty-one markets during the summer. October has a low return for

[7] Note that FF use factor models with both *HML* and *HMLO*, a statistically modified factor. They only display factor loadings for the *HMLO* version of their models.

[8] We prefer to add the *R* subscripts to indicate a return-based factor.

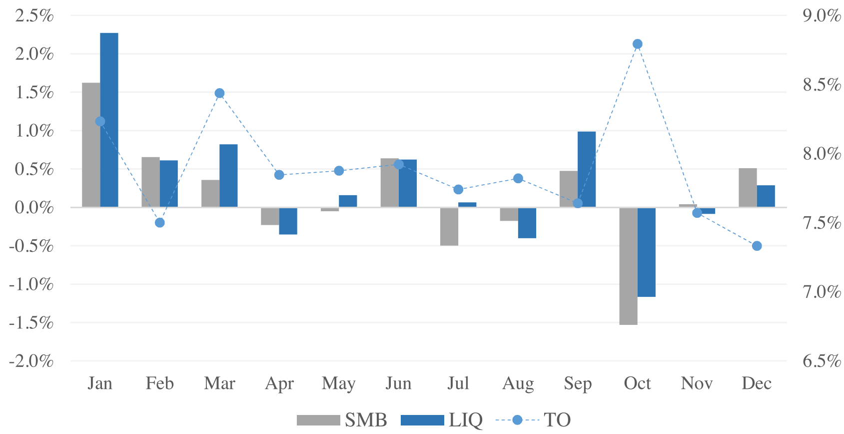

Fig. 1. ***LIQ*$_R$ and *SMB*$_R$** Average Factor Premium by Month. This figure shows average returns by month for two factors over the time period 1969 - 2017. *SMBR* is the Fama and French factor for size and *LIQR* is our liquidity factor. *TO*, differently scaled on the right, is average monthly turnover on the NYSE calculated as per Section 2.6.

the SMB_R and LIQ_R factors: −1.53 % and −1.17 % per month, respectively. Turnover (*TO*) is shown as well; it is higher in January, March and October. The Tax Reform Act of 1986 requires mutual funds to realize capital gains during the twelve-months ending October 31, see Haug and Hirschey (2006) on this and for an update on Ritter (1988). Relating the rich calendar anomaly literature to these models is something we hope to study, but we believe the calendar phenomenon are linked to liquidity, behavior, or sentiment; not size.

While LIQ_R may be a consumption motivated factor related to, but not perfectly aligned with a firm risk size factor, others are skeptical of the SMB_R factor. SMB_R has a well-known payoff drought since its discovery in the 1980's. If we split our data period in half: the average SMB_R monthly return for 1968−02 to 1993−03 is 0.17 % (t-stat = 0.99) and for 1993−04 to 2018−05 it is 0.20 % (t-stat = 1.12) per month. For LIQ_R it is 0.27 % (t-stat = 1.72) and 0.38 % (t-stat = 2.34) for these same periods, respectively.[9] Alquist, Israel, and Moskowitz (2018) point out that there is no size premium in corresponding issuers' corporate bonds, and Crain (2011) summarizes skeptical literature of the size premium.

Earnings are closely related to *HML*, net of stock issuance retained earnings account for changes in book value. Fama and French (1996, 2006) also discuss this relationship as well as the relationship of *HML* to firms with negative earnings. In our view, earnings are a way to measure the concern of investors of hedging their earnings and consumption risk. Earnings, dividends, or cash flow have been modelled as a risk in consumption related models by Merton (1971); Abel (1999) and in Snigaroff and Wroblewski (2020).

We include negative earnings which do not provide measurement problems in our factor model. Importantly, E/P_R is a factor with purer exposure to earnings risk; if investors are concerned about risk to investment earnings when their own human capital earnings are at risk as during a recession, a high loading on E/P_R causes their portfolio to perform worse at exactly the wrong time, hence an E/P_R factor premium.[10] Fama and French (2015) add a type of profitability, the profits-to-assets factor which has also been proposed by Hou et al. (2015) as being a production, or Q-theory factor. Rather than mix production with demand factors, our higher dimension models include differently measured factors from the same characteristics that other researchers cited here relate to investor's concerns.

3. ELM model tests

3.1. Redundancy test

To evaluate if our model's factors are all required; we regress our model's factor time series on the remaining model factors over the time-period of our study 1968–2018. Table 5 shows results for the ELM model. Our ELM factor model has four statistically significant intercepts at 95 % confidence (intercepts in our model testing section follow the generic "a" nomenclature). ELM has materially greater significance on the **a**'s for LIQ_R, E/P_R and $LIQG_R$, than Snigaroff and Wroblewski's (2018) results (three significant factors, all with lower coefficients).[11] Table 5 shows EG/P_R is subsumed by other factors. The most significant loading is on $LIQG_R$; we would expect that companies with high earnings growth generate an increase in investor trading. In the next section we show both four and five factor versions of our model work well in asset pricing tests. For the non MKT_R factors ELM's largest pairwise loadings are the 0.57 coefficient (t-stat of 10.53) for $LIQG_R$ on EG/P_R.

3.2. Asset pricing tests

Gibbons, Ross, and Shanken (1989) developed what is now known as the GRS statistic to test the null hypothesis that all the intercepts of N time series regressions are simultaneously zero: H_0: $a_i = 0; i = 1,2,3, \ldots, N$. We regress test portfolio returns against competing factor models and compute this test statistic. Ideally, we want the intercepts to equal zero as they are a measure of a model's pricing error. Non-rejection of the null hypothesis indicates model success and displays statistical evidence of a given model's ability to span the return space. The nearer all the intercepts are to zero, the greater the ability of the model to price the test assets under consideration. Therefore, we prefer a model with a low GRS statistic and a high p-value. We also follow Barillas and Shanken (2017) and Fama and French (2018) in computing a measure of variation. We denote by $\bar{r}_i$ as the *i-th* portfolio's deviation of return from the cross–sectional average return of all the portfolios and use this measure in the ratios that describe model variation in Table 6.

[9] Fama and French (2012) use 1963−2013 data to obtain a 0.29% *SMB* mean with 2.31 t-stat.

[10] This is the risk factor view. The behavioral hypothesis for a value premium of Lakonishok, Shleifer, and Vishny (1994) includes a rationale of individual investor bias and their study uses measures with "multiples of price to current earnings and cash flow" (p. 1543) as well as book-to-market.

[11] We do not show it, but for a similar time period FF has significant **a**'s for SMB_R, RMW_R, and CMA_R.

Table 5
Factor Spanning Tests: Within Model.

ELM on ELM Factor Subsume Tests, 1968–2018
Left column factor regressed on all of the other factors.

		a	MKT_R	LIQ_R	E/P_R	EG/P_R	$LIQG_R$	R^2
MKT_R	Coef	0.88		−0.21	−0.90	0.47	−0.21	39.22
	t-stat	*4.85*		*−3.19*	*−18.42*	*5.49*	*−3.47*	
LIQ_R	Coef	0.45	−0.08		0.02	−0.05	−0.13	5.60
	t-stat	*3.94*	*−3.19*		*0.63*	*−0.95*	*−3.41*	
E/P_R	Coef	0.43	−0.40	0.03		0.41	−0.04	40.37
	t-stat	*3.50*	*−18.42*	*0.63*		*7.21*	*−0.98*	
EG/P_R	Coef	0.00	0.10	−0.03	0.20		0.27	23.52
	t-stat	*−0.06*	*5.49*	*−0.95*	*7.21*		*10.53*	
$LIQG_R$	Coef	0.54	−0.09	−0.15	−0.04	0.57		19.38
	t-stat	*4.39*	*−3.47*	*−3.41*	*−0.98*	*10.53*		

We regress the ELM five-factor model's factors separately against the complementary four factors and a constant in that particular model. Table 5 shows the ELM model factors against the ELM complementary factors; a are intercepts.

Table 6
GRS tests.

6-A: ELM Factors Summary Statistics on the GRS Test

Model Factors	GRS	GRS pvalue	$A\|a_i\|$	$\frac{A(a_i^2)}{A(\bar{r}^2)}$	$\frac{A(s^2(a_i))}{A(a_i^2)}$	$A(R^2)$
Panel A: 25 LIQ-E/P portfolios						
LIQ_R E/P_R	2.02	0.002	0.097	0.35	0.44	0.89
LIQ_R E/P_R EG/P_R	1.95	0.004	0.099	0.36	0.43	0.89
LIQ_R E/P_R $LIQG_R$	1.66	0.024	0.086	0.32	0.49	0.89
LIQ_R EG/P_R $LIQG_R$	2.16	0.001	0.190	1.21	0.18	0.85
LIQ_R E/P_R EG/P_R $LIQG_R$	1.66	0.024	0.086	0.32	0.49	0.89
Panel B: 25 LIQ-EG/P portfolios						
LIQ_R E/P_R	2.68	0.000	0.100	0.42	0.40	0.88
LIQ_R E/P_R EG/P_R	2.53	0.000	0.085	0.33	0.44	0.90
LIQ_R E/P_R $LIQG_R$	2.91	0.000	0.092	0.43	0.39	0.89
LIQ_R EG/P_R $LIQG_R$	2.89	0.000	0.115	0.46	0.33	0.89
LIQ_R E/P_R EG/P_R $LIQG_R$	2.90	0.000	0.092	0.43	0.34	0.90
Panel C: 25 LIQ-LIQG portfolios						
LIQ_R E/P_R	3.82	0.000	0.252	0.94	0.09	0.85
LIQ_R E/P_R EG/P_R	3.67	0.000	0.223	0.76	0.10	0.86
LIQ_R E/P_R $LIQG_R$	2.85	0.000	0.114	0.21	0.29	0.89
LIQ_R EG/P_R $LIQG_R$	2.88	0.000	0.120	0.23	0.28	0.88
LIQ_R E/P_R EG/P_R $LIQG_R$	2.85	0.000	0.114	0.21	0.29	0.89

6-B: FF (2015) Factors Summary Statistics on the GRS Test

Model Factors	GRS	GRS pvalue	$A\|a_i\|$	$\frac{A(a_i^2)}{A(\bar{r}^2)}$	$\frac{A(s^2(a_i))}{A(a_i^2)}$	$A(R^2)$
Panel A: 25 LIQ-E/P portfolios						
SMB_R HML_R	2.87	0.000	0.166	0.99	0.23	0.85
SMB_R HML_R RMW_R	2.39	0.000	0.123	0.66	0.31	0.86
SMB_R HML_R CMA_R	3.15	0.000	0.179	1.13	0.20	0.85
SMB_R RMW_R CMA_R	2.48	0.000	0.137	0.77	0.30	0.85
SMB_R HML_R RMW_R CMA_R	2.63	0.000	0.142	0.81	0.26	0.86
Panel B: 25 LIQ-EG/P portfolios						
SMB_R HML_R	3.57	0.000	0.179	1.08	0.20	0.85
SMB_R HML_R RMW_R	3.51	0.000	0.168	0.98	0.21	0.86
SMB_R HML_R CMA_R	3.84	0.000	0.184	1.13	0.19	0.85
SMB_R RMW_R CMA_R	3.85	0.000	0.178	1.08	0.20	0.86
SMB_R HML_R RMW_R CMA_R	3.94	0.000	0.180	1.10	0.19	0.86
Panel C: 25 LIQ-LIQG portfolios						
SMB_R HML_R	4.55	0.000	0.269	1.16	0.08	0.84
SMB_R HML_R RMW_R	4.19	0.000	0.236	0.98	0.09	0.84
SMB_R HML_R CMA_R	4.07	0.000	0.251	1.02	0.09	0.84
SMB_R RMW_R CMA_R	3.62	0.000	0.210	0.81	0.12	0.84
SMB_R HML_R RMW_R CMA_R	3.64	0.000	0.208	0.80	0.12	0.84

This table compares the ability of the ELM model (6-A) and the FF model (6-B) to span the space of test portfolio returns. In the leftmost column of each panel we list the factors used for that row's statistics. The header of each panel denotes the partitioning structure of the twenty-five test portfolios. For example, in panel A of the ELM (6-A) we form the test portfolios based on the intersections of the two univariate sorts *LIQ* and *E/P*. We then regress the twenty-five quintile intersection portfolios against the factors listed in the first column. MKT_R is a factor in every model shown but not listed. We also note that in 6-B, MKT_R, denotes MKT_R^{CAP}. The GRS–statistic and its corresponding *p*-value that arise from the *F*-test are displayed in columns two and three respectively. Column four is the average over the absolute values of each of the twenty-five portfolio regression intercepts: a measure of the error of the corresponding factor model and thus lower is better. Column five is the ratio of average squared intercept to the average square of each portfolio returns deviation from the cross-sectional average over these portfolio returns. This is a measure of dispersion relative to the average dispersion and a low ratio is preferred. Column six is the ratio of the average over the intercept estimate sample variance relative to the average square of these intercepts. High is better. Finally, column seven reports the average over the adjusted *r-squared* statistic.

Table 6 shows statistics for the asset pricing factors based on the described partitions labeled in the leftmost column. Panel A is formed by partitioning the universe based on the intersections of the two univariate sorts *LIQ* and *E/P*. We then regress the twenty-five quintile intersection portfolios against either the three, four, or five factors listed in this first column.

Column two and three report the GRS–statistic and the corresponding *p*-value that arises from the hypothesis test described above. Column four has the average of the absolute values of each of the twenty-five portfolio regression intercepts. Column five displays the ratio of average squared intercept to the average square of each portfolio returns deviation from the cross-sectional average over these portfolio returns. This is a measure of dispersion relative to the average dispersion. A low ratio is preferred as this may be viewed as the percentage of left hand-side return dispersion attributed to the intercept dispersion. The sixth column is the ratio of the average over the intercept estimate sample variance relative to the average square of these intercepts. We would like this to be high as it says that much of the dispersion in the intercepts is attributed to sampling error rather than from true intercept dispersion. Finally, column seven reports the average over the adjusted r-squared statistic corresponding to each of the regressions. As a comparison, Panel B reports these same measures for the different combinations of the FF factors using the same test portfolios.

For most of the GRS tests shown in Table 6, we must reject the null hypothesis of no pricing errors and the notion of a perfect factor basis for the return space at the 95 %, and 99 % levels. Interestingly, when test portfolios are sorted on *LIQ* and *E/P* we find that two of the models are not rejected at a 99 % level of confidence. This is the case for the five factor model of (1), as well as the four factor model shown in (2). Both have fairly low GRS statistics of 1.66 with *p*-values of 0.024. It is a rarity for these kinds of models to not be rejected.

$$R_{i,t} - R_{F,t} = \alpha_i + \beta_i \cdot MKT_{R,t} + \phi_i \cdot LIQ_{R,t} + \pi_i (E/P)_{R,t} + \gamma_i \cdot LIQG_{R,t} + \varepsilon_{i,t}. \quad (2)$$

Table 7
Cross-Sectional R-Squared Tests, Difference in Sample R-Squared Statistics.

	GLS		OLS	
	ELM–FF	*P-Value*	ELM–FF	*P-Value*
Panel A: ELM and FF Sequential Non-Nested				
ELM Five-Factors – FF Five-Factors	0.057	0.616	−0.028	0.112
Panel B: ELM Nested				
ELM factors: MKT_R LIQ_R E/P_R EG/P_R $LIQG_R$	ELM	*P-Values*	ELM	*P-Values*
MKT_R	**0.190**	0.041	0.487	0.067
MKT_R LIQ_R	**0.178**	0.033	0.091	0.461
MKT_R LIQ_R E/P_R	**0.174**	0.015	0.083	0.248
MKT_R LIQ_R E/P_R $LIQG_R$	**0.113**	0.021	0.081	0.120
MKT_R LIQ_R E/P_R EG/P_R	**0.164**	0.007	0.072	0.089
Panel C: FF Nested				
FF factors: MKT_R SMB_R HML_R RMW_R CMA_R	FF	*P-Values*	FF	*P-Values*
MKT_R	0.100	0.105	**0.524**	0.023
MKT_R SMB_R	0.024	0.608	0.134	0.216
MKT_R SMB_R HML_R	0.003	0.909	0.029	0.457
MKT_R SMB_R HML_R RMW_R	0.003	0.707	0.008	0.463
MKT_R SMB_R HML_R CMA_R	0.000	0.980	0.027	0.266

GLS refers to generalized least squares and OLS refers to Ordinary least squares. In Panels A–C column 2–3 are GLS and columns 4–5 are OLS. This table displays the difference in cross-sectional *r-squared* statistics. Panel A shows the non-nested model comparisons for both the GLS and OLS cases. Panel B shows the nested model cases for the ELM five-factor model and Panel C shows the nested Fama-French five-factor model results. We denote in **bold** the statistics that are significant at the 5% level.

Also, in all fifteen of the cases presented the ELM model has a lower GRS-statistic than the competing FF model to its right. However, the test portfolios are sorted on our characteristics, not theirs. That is, Table 6 is not meant as a 'horse-race' between models but as indication that ELM compliments the FF factor description. The adjusted *r-squared* averages are higher in fourteen of fifteen cases and the average magnitude of the intercepts is lower in all but one case. Columns five and six also demonstrate competitive measures of dispersion. These diagnostic results show that very competitive three, four and five factor models can be formed with our variables. Earnings and liquidity factors' ability to span the return space with small error is attractive as these are characteristics investors have followed closely.

As another robustness test, we consider a cross-sectional goodness of fit test in Table 7. We denote by R the matrix of test portfolio returns consisting of the twenty-five size-B/M series of returns along with five industry portfolio series of returns. These may be found on the Kenneth R. French[12] website. The construction of the hypothesis tests with the use of these test portfolios, follow (Kan, Robotti, and Shanken (2013), Table IV) in the context of the ELM and FF models as a further goodness of fit and model comparison test. A more detailed summary of this test may be found in Appendix A. We present both the *OLS* and the *GLS* versions of the difference in cross-sectional R-squared statistics with their p-values. We consider both the nested and non-nested model cases. A model is nested into a second model if all the first model's factors are contained as factors in the second model. We provide the *p-values* (in which smaller implies more evidence of model differences) under the assumption that the models may be mis-specified, and we use the sequential tests for the non-nested model cases. The non-nested model case is shown in Panel A of Table 7. For both the *GLS* and the *OLS* cases we do not find enough statistical significance to show that the difference in cross-section *r-squared* statistics are different from zero. In this sense the ELM and FF somewhat 'tie' in cross-sectional performance. Panel B displays the difference in the sample cross-sectional *r-squared* statistics between the ELM model and the nested factor model listed in the first column. Panel C shows the analogous statistics for the FF model. Significant (at the 5% level) cross-sectional *r-squared* statistics are labeled in bold, which in this case is preferred. In the *GLS* case the ELM model significantly outperforms the cross-sectional *r-squared statistic* of each of the nested models. In the OLS case the ELM model only significantly improves on the MKT_R (CAPM) nesting. Panel C shows the FF model also improves its *r-squared* over its nested models, however, the improvement is not significant in terms of improving the cross-sectional *r-squared* against its nested models labeled in column one. This is not a test of one model compared to the other, indeed, FF also add a statistical version of their *HML* factor to improve model fit. In our search for a descriptive basis to span the return space it is attractive that the intuitive characteristics we use do well in this test. Table 7 (and Tables 5, and 6) show each of these characteristics, despite their simplicity and internal relatedness, generally contribute to the model's descriptive power.

3.3. Anomaly spanning tests

We now compare the ELM and FF models by running anomaly spanning tests on the eight anomalies listed and described in Appendix B. Eight anomalies are a small sample of the hundreds possible. Nevertheless, it is a list of factors used by Fama and French (2008, 2016) and provided on Kenneth R. French's webpage. To avoid selection bias, we test against the eight factors featured in these two papers (there is some overlap). Note that even though the Fama and French (2015) model was based in a fundamental valuation argument, part of the motivation of that work was to subsume more anomalies. Four of these eight anomalies are the same characteristic used to form their factors whereas none are the same characteristic used for our five factors (we use a closely related market factor weighted on liquidity instead of their capitalization). All that to say, this list should be a more difficult set of anomalies for ELM.

The anomalies are calculated via return spreads for decile opposite portfolios. We regress the decile returns for each of these eight anomalies on the model factors for the two models. In Table 8 we report the GRS, the GRS p-value, and the average of the absolute value of the intercepts – $A|a_i|$. Another statistical comparison of the ELM model's ability to span the anomaly space is a measure given in Fama and French (2018) that again involves the intercepts obtained from the ten decile time series regressions. This measure is the *maximum squared Sharpe ratio for the intercepts* (SRI) and is defined as:

$$Sh^2(a_i) = a_i^T \Sigma_i^{-1} a_i, \tag{3}$$

[12] https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

Table 8
Spaning the Anomaly Space.

Puzzle	Model	GRS	GRS pvalue	$A\|a_i\|$	SRI
B/M	ELM	0.92	0.51	0.06%	0.02
	FF	1.51	0.13	0.09%	0.03
Operating Profitability	ELM	2.76	0.00	0.10 %	0.05
	FF	1.21	0.28	0.05%	0.02
Investments	ELM	2.69	0.00	0.08%	0.05
	FF	2.55	0.01	0.08%	0.05
Momentum	ELM	2.40	0.01	0.11%	0.04
	FF	3.84	0.00	0.26%	0.07
Accruals	ELM	4.20	0.00	0.12 %	0.08
	FF	4.86	0.00	0.13%	0.09
Market Beta	ELM	2.23	0.01	0.12 %	0.04
	FF	2.50	0.01	0.10 %	0.05
Net Share Issues	ELM	4.91	0.00	0.13%	0.09
	FF	3.71	0.00	0.14 %	0.07
Variance	ELM	5.70	0.00	0.15%	0.11
	FF	3.10	0.00	0.11%	0.06

Regressions of decile returns for each of these eight anomalies on the model factors for the two models, ELM (five-factor) and FF (five-factor) of Fama and French (2015). We report the GRS, GRS pvalue, the average of the absolute value of the intercepts – $A|a_i|$, and the Sharpe Ratio for the intercepts – SRI.

where the vector, a, represents the intercepts from the regressions against factor model i, and $\sum_i$ is the ten-by-ten covariance matrix for the residuals. This number represents how close we are to spanning the anomaly space of returns with our factor model, hence lower is better due to less error. We show the Maximum Sharpe Ratio for the intercepts across the different anomalies in column six of Table 8. The ELM model has a smaller maximum squared Sharpe ratio for the intercepts, and better values for all the measures, in four out of the eight cases relative to the FF model. While ELM beats FF four of eight times in this test against anomalies, ELM has difficulties with seven (large GRS) while FF has difficulties with six.

The ELM model has problems with all of the anomalies except B/M, while FF have trouble with all of the anomalies except for B/M and Operating Profitability. Fama and French (2016) provide helpful coverage of the issues these anomalies present. Our comparison of these anomalies is not to say one model is preferred, but to demonstrate that with the anomalies they select, ELM does well, and can provide an alternative and powerful description of the cross-section. Both models have difficulty with the Momentum anomaly, with ELM doing significantly better. Regarding ELM's difficulty here, note that we construct our $LIQG_R$ factor differently than MOM_R. The latter leaves out the most recent one-month return, $LIQG_R$ does not leave out the most recent one-month change in volume. The momentum decile anomaly is the significant problem for both models, and the momentum factor is discussed in the next sub-section. Note that if we replace $LIQG_R$ with MOM_R (the Carhart (1997) momentum factor is described in the next section) in equation (2) and attempt to span the Momentum deciles anomaly space by computing the statistics of Table 8 it underperforms the ELM model.

3.4. Momentum

The large premium, persistence across time and markets, e.g., Asness et al. (2013), and the difficulty justifying in rational pricing, makes momentum an embarrassing anomaly for financial economists with a goal of describing rational actors setting prices. Although Johnson (2002, p.585) argues momentum "need not imply investor irrationality" Fama and French (2018, p.237) frankly warn about their own inclusion of momentum; "We worry, however, that opening the game to factors that seem empirically robust but lack theoretical motivation has a destructive downside: the end of discipline that produces parsimonious models and the beginning of a dark age of data dredging..." As motivation we have assumed that investors are concerned about earnings and liquidity which is a 'rational pricing' of risk view. Other behavioral models include volume as an indicator for sentiment, examples including Barberis, Shleifer, and Vishny (1998); Baker and Stein (2004) and Baker and Wurgler (2006). Sentiment is not disjoint with 'rational investor behavior' as volume and changes in volume can be an indicator for a well-functioning market, our assumption here. Our variables for LIQ_R and $LIQG_R$ are related to Lo and Wang's (2006) theory of volume and their "portfolio that is used to hedge the risk of changing market conditions" [p. 2805]. Gervais et al. (2001) and Blume et al. (1994) relate volume to equilibrium while Lee and Swaminathan (2000) and Hong and Stein (2007) study trading volume and price momentum, with the latter endogenizing sentiment to explain high volume levels.

Jegadeesh and Titman (2011, p.505) and Daniel and Moskowitz (2016)[13] consider momentum crashes. The latter give various reasons for possible momentum crashes but do not mention liquidity. As they explain, "when the market has fallen significantly over the momentum formation period (in our case from 12 months ago to one month ago) a good chance exists that the firms that fell in tandem with the market were and are high-beta [beta on momentum] firms, and those that performed the best were low-beta firms. Thus, following market declines, the momentum portfolio is likely to be long low-beta stocks (the past winners) and short high-beta stocks (the past losers)" [p. 222]. But this is exactly what we would expect through a liquidity crisis. Firms with highest liquidity exposures fall the most, and during recovery a rebalanced high-low $LIQG_R$ factor would then be long the low γ stocks of Table 4 and short high γ stocks and would naturally suffer during the recovery. Examining the time series properties of our factors are left to a future study, but we can see the relation of MOM_R with our factor $LIQG_R$, via their 0.89 correlation from 1968–2018. This is consistent with the financial intermediation literature; markets 'malfunction,' have 'panics,' or 'liquidity spirals.'

We assign MOM_R as the name for the Carhart (1997) momentum factor that is frequently used by researchers and is the series from Kenneth R. French's website. For our time-period 1968–2018 the mean monthly return for MOM_R is 0.65 % with a t-stat of 3.73. Table 9 Panel A reports the results for MOM_R regressed on the ELM

[13] Bohl, Czaja, and Kaufmann, (2016)) also study momentum crashes in Germany. They find earnings momentum (what we call EG/P_R) growth largely escape market recoveries when momentum crashes and, similar to others, note its relation to momentum. We find $LIQG_R$'s relation to momentum is far higher.

Table 9
Momentum Factor Spanning Tests.

Panel A: MOM_R regressed on ELM factors.

	a (%)	MKT_R	LIQ_R	E/P_R	EG/P_R	$LIQG_R$	R^2
Coefficient	0.10	−0.09	−0.13	−0.13	0.14	1.12	81.34
t-stat	*1.26*	*−5.32*	*−4.45*	*−5.03*	*3.61*	*43.35*	

Panel B: MOM_R regressed on FF factors.

	a (%)	MKT_R^{CAP}	SMB_R	HML_R	RMW_R	CMA_R	R^2
Coefficient	0.71	−0.13	0.02	−0.57	0.22	0.46	10.12
t-stat	*4.05*	*−3.08*	*0.31*	*−6.97*	*2.69*	*3.75*	

Panel C: $MOM1Y_R$ regressed on ELM factors.

	a (%)	MKT_R	LIQ_R	E/P_R	EG/P_R	$LIQG_R$	R^2
Coefficient	−0.12	−0.10	−0.14	−0.15	0.12	1.10	79.84
t-stat	*−1.48*	*−5.70*	*−4.68*	*−5.64*	*2.98*	*41.39*	

Panel D: $MOM1Y_R$ regressed on FF factors.

	a (%)	MKT_R^{CAP}	SMB_R	HML_R	RMW_R	CMA_R	R^2
Coefficient	0.47	−0.14	0.00	−0.59	0.17	0.48	10.75
t-stat	*2.74*	*−3.28*	*0.04*	*−7.34*	*2.18*	*3.98*	

The factor, MOM_R has a mean of 0.65 with t-stat of 3.73. $MOM1Y_R$ has a mean of 0.40 and a *t*-stat of 2.30. We regress either MOM_R or $MOM1Y_R$ on ELM or FF five-factor models as indicated; a are intercepts.

model factors. The intercept falls to 0.10 % with a *t*-stat of 1.26, i.e., MOM_R is subsumed. This compares to the Fama and French (2015) factor model (which we again refer to as FF throughout this section) intercept in Panel B of 0.71, an increase, and a *t*-stat of 4.05. Interestingly, there are materially different loadings on E/P_R, −0.13, and −0.57, for HML_R. Crucially, the coefficient on $LIQG_R$ is 1.12 with a *t*-stat of 43.35. The relationship is one-for-one and powerfully significant.

MOM_R is a composite factor. It includes one-year momentum but 'nets out' the one-month reversal of Jegadeesh (1990) by only using return months *t − 12* to *t − 2* (eleven months), in factor formation. One-month reversal is postulated as a liquidity phenomenon, see Jegadeesh and Titman (2011). To dissect our ability to subsume momentum, we build a one-year momentum variable $MOM1Y_R$ as per Jegadeesh and Titman (1993). It is constructed in the same way as MOM_R except that it includes the most recent month in the prior return sort and thus uses the trailing twelve-month returns. Panel C shows $MOM1Y_R$ regressed on the ELM factors, the intercept is −0.12 % with a −1.48 *t*-stat; the ELM model also does a good job minimizing one-year momentum.

Blume et al. (1994) propose theory for volume being convex in price for the well-known empirical phenomenon of a strong relation between the absolute value of price changes and volume. Large price changes either up or down, are associated with large volume. We constructed our $LIQG_R$ factor to separate volume that is associated with good news about a company from volume associated with bad news in the following way: $LIQG_t := (\$Volume_t - \$Volume_{t-12}) / \$Volume_{t-12}$ when the cumulative return, from time *t-12* to time *t-1* (eleven months), for a stock is greater than or equal to zero. When the cumulative return, from time *t-12* to time *t-1*, is less than zero $LIQG_t := -1 \times |(\$Volume_t - \$Volume_{t-12}) / \$Volume_{t-12})|$. This converts the convex (v-shaped) relationship between volume growth and returns to be 'linear' as it should be. This is preferred as the relationship between liquidity growth and news about that firm's 'true' exposure to liquidity growth is contaminated by temporary large volume selling events associated with 'bad news.' If changes in liquidity are a systematic risk exposure, then we need to properly sign idiosyncratic bad news events that distort this risk. We also constructed a model version that weeded out stocks that do not have a positive total return (returns less than or equal to zero) in the past month to form our universe to build the $LIQG_R$ factor without the bad news stocks. Such a model also subsumes MOM_R but we opted for the more inclusive of the universe factor definition. We believe momentum is largely a result of investor's concern with a time-varying state of liquidity risk and these results are encouraging for our view.

An alternative view is that our calculation of $LIQG_R$ in Table 1 is very similar to a calculation for momentum, hence $LIQG_R$ is merely a repackaging of momentum. As can be seen in Table 1, the characteristics are mechanically linked. However, other researchers cited earlier have argued for a changing state of liquidity as a risk factor. If $LIQG_R$ is indeed an effective empirical proxy for this state, it should not be surprising that momentum was 'discovered' as an anomaly. We should expect their relation; as Pontiff and Singla (2019, p. 272) point out, "a necessary condition for all theories of illiquidity is a contemporaneous relation between absolute returns and volume." Further, momentum may be a behavioral driven factor in addition to a changing liquidity state as modeled by $LIQG_R$, which is an interesting area of continuing research.

4. Conclusion

Our cross-sectional model uses levels of obvious stock characteristics along with their corresponding rates of changes in order to explain stock returns. While other researchers have made the case that the characteristics we use; the market, earnings, and liquidity are state risk candidates, it may be the case that these are characteristics that proxy for 'true' but unknown state variables. Or it may be the ELM model effectively isolates investor behavior with large liquidity premiums offering evidence of irrationality. In any event, the inclusion of liquidity in describing stock returns in a candidate benchmark factor model benefits construction by addressing long held investor interest in volume and the vast liquidity literature.

Apart from the Market, a well-known factor since 1964, we accomplish our results with simple, well-known variables that investors have cared about for over one-hundred years. Ironically, earnings' and volumes' importance were well known by investors long before *HML* and *SMB* became known by academics. It is difficult to make a data mining or p-hacking critique against characteristics regularly reported for so long, or on our standard model design. Hou et al. (2017) report at least 447 different proposed anomalies and their many different combinations, with researchers' latitude in model construction, makes it highly likely that marginal improvements in factor models will be obtained. Multi-factor models that are not based on first principles and obvious investor concerns require a higher standard.

Muir's (2017) evidence is inconsistent with sentiment, but consistent with models that incorporate an institutional or credit view, and others cited earlier have argued for a central role for liquidity. Our placement of liquidity as a crucial part of asset prices is consistent with the importance of financial intermediation. Whether or not investors are rational regarding the size of the liquidity premiums, the differentiation of liquidity and sentiment, whether investors are rationally forward looking, or that equilibrium includes a role for intermediation are interesting areas of continuing research. But liquidity, as proxied by volume, is an important facet of investor's concern. We show this in a well working cross-sectional model that can subsume momentum.

Declaration of Competing Interest

Both authors work at an investment advisory firm, Denali Advisors. The firm does not offer factor investment or ETF products. The firm is not expected to profit from publication of this research beyond general prestige.

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Appendix A. CROSS-SECTIONAL GOODNESS OF FIT TESTS

Let R indicate the matrix of test portfolio returns and denote by F the matrix of factor returns and define the covariance matrix for the factor returns and the test portfolio returns, with the covariance matrix of the factors alone represented by V_{RF}, and V_F respectively. These tests are constructed by using the cross-sectional R-squared statistics (CSR). One may consider a two-pass cross-sectional regression to construct this measure. The construction begins with a beta pricing model given by:

$$\mu_R = X\eta, \tag{A1}$$

where μ_R denotes the mean of the test portfolio returns and the matrix X contains a vector of ones and the betas from a time series regression of the test portfolios onto the factors:

$$R_t = \alpha + \hat{\beta} \cdot F_t + \xi_t, \quad t = 1, 2, 3, \ldots T. \tag{A2}$$

This gives a vector of betas for each test portfolio. The second pass of the regression is to use the matrix $X = \left[\vec{1}_N | \ \hat{\beta} \right]$ with $\vec{1}_N$ being a vector of ones with length N, the number of test portfolios. Since the betas are given by a multivariate regression one may use $\hat{\beta} = \hat{V}_{RF} \cdot \hat{V}_F^{-1}$, see the internet appendix[14] of Kan et al. (2013) for more details. We consider two forms of this second estimation which are both specified by a symmetric weighting matrix W. For the *OLS-CSR* case, W is the identity matrix and for the *GLS-CSR* case $W = V_R^{-1}$. When we use (A1) as a pricing model and this two-pass methodology in order to estimate η we obtain the asset pricing error of our test assets:

$$\varepsilon_W = \mu_R - X\hat{\eta} = \left(I_N - X(X'WX)^{-1}X'W \right) \mu_R. \tag{A3}$$

[14] Appendix: http://www-2.rotman.utoronto.ca/-kan/research.htm.

The paper of Kandel and Stambaugh (1995) then defines the sample cross-sectional R-squared measure as:

$$\rho_W^2 = 1 - \frac{\varepsilon'_W W \varepsilon_W}{\varepsilon'_0 W \varepsilon_0}, \tag{A4}$$

where $\varepsilon_0 = \left(I_N - \vec{1}_N \left(\vec{1}'_N W \vec{1}_N \right)^{-1} \vec{1}'_N W \right) \mu_{R,}$ which represents the deviations of the mean returns from their cross-sectional average. Since W is symmetric, we can also factor part of this expression to obtain:

$$\begin{aligned} \varepsilon'_W W \varepsilon_W &= \mu'_R \left(I_N - W'X(X'WX)'^{-1}X' \right) W \left(I_N - X(X'WX)^{-1}X'W \right) \mu_R \\ &= \mu'_R W \mu_R - \mu'_R W'X(X'WX)^{-1}X'W\mu_R. \end{aligned} \tag{A5}$$

One further simplification, that is given in Kan et al. (2013), is the fact that one obtains the exact same pricing errors, ε_W, as they would by estimating in the way as described above by using $\tilde{X} = \left[\vec{1}_N | \ \hat{V}_{RF} \right]$, in place of the prior matrix $X = \left[\vec{1}_N | \ \hat{V}_{RF} \cdot \hat{V}_F^{-1} \right]$. This may be seen by defining the invertible matrix $C = \begin{pmatrix} 1 & \vec{0}_{K'} \\ \vec{0}_K & V_F^{-1} \end{pmatrix}$, and noticing that $\tilde{X}C = X$. Therefore any solution $\hat{\eta}$, of (A1), is also a solution of the analogous equation involving $\tilde{X}$ by using $C\hat{\eta}$. Similarly since C is invertible any $\hat{\lambda}$ solution corresponding to $\mu_R = \tilde{X}\lambda$ is also a solution of (A1) with $\hat{\eta} = C^{-1}\hat{\lambda}$. This leads to the hypothesis test in which we consider both the nested models and the non-nested models cases and use the difference in the sample cross-sectional R-squared statistics given in (A4) to consider:

$$\begin{aligned} H_0 &: \rho_i^2 = \rho_j^2 \\ H_1 &: \rho_i^2 \neq \rho_j^2, \end{aligned} \tag{A6}$$

to test the differences between model i and model j. Fortunately, the asymptotic distribution for the difference in sample $\hat{\rho}_W^2$ statistics and the other test statistics have been computed for us in Kan et al. (2013). For the nested model cases we test the difference in sample *r-squared* statistics and linear combination of *chi-squared* distribution. In the non-nested model cases we use a sequential test based on first testing if the normalized *SDF*'s are equal which uses a *chi-squared* test. If we are not able to reject the equality of the normalized *SDF*'s test, then we may use that *p-value* for our hypothesis in (A6) since equal *SDFs* would imply equal *r-squared* statistics. If we reject the equal *SDF*'s case, then we test for both model's equality of *r-squared* statistics with proper specification for both models. If that test is not rejected, we may then use this *p-value* from the properly specified model *chi-squared* test in order to interpret (A6). If both tests are rejected, then we evaluate our null hypothesis under a normal distribution assumption for the difference in *r-squared* test statistic.

Appendix B ANOMALY DATA

Brief definitions of the decile portfolios, the returns of which are from French's website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html. The market value of equity (ME) is the price times the number of shares outstanding. Breakpoints for the FF deciles are calculated with NYSE stocks. Stocks are rebalanced at the end of June of t for July of year t to June of t + 1 or monthly.

Deciles	Rebalancing	Definition
B/M	June-end annually	Book value of equity (BE) for the last fiscal year end in t-1; ME at December-end in t-1; for all stocks also to have ME for December.
Operating Profitability	June-end annually	(Sales – COGS – Interest expense – S,G & A expenses)/BE for the last fiscal year end in t-1.
Investments	June-end annually	(Total assets (TA) in t-1 minus TA in t-2)/TA in t-2.
Momentum	End of month t-1 monthly	Monthly NYSE prior (2−12) return decile breakpoints; all stocks having prior return data with a price at t-13 month-end and a good return for t-2; ME for t-1 month-end.
Accruals	June-end annually	Accruals for June of t are the change in operating working capital per split-adjusted share from the fiscal year end t-2 to t-1 divided by book equity per share in t-1. NYSE breakpoints and portfolios all stocks having ME for June of t, (positive) book equity data for t-1, and operating working capital data for t-2 and t-1.
Market Beta	June-end annually	Univariate market beta for year t is estimated using the preceding five years (two minimum) of past monthly returns. NYSE breakpoints and all stocks having ME for June of t and good returns for the preceding 60 months (24 months minimum).
Net Share Issues	June-end annually	Net Share Issues NI < 0; NI = 0; NI for June of t is the change in the natural log of split-adjusted shares outstanding from the fiscal yearend in t-2 to the fiscal yearend in t-1. NYSE breakpoints and portfolios all stocks having ME for June of t and split-adjusted shares outstanding data for t-2 and t-1.
Variance	End of month t-1 monthly	Variance of daily returns is estimated using 60 days (minimum 20) of lagged returns. NYSE breakpoints and portfolios all stocks with ME for the end of month t-1 and an existing variance.

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