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THE SUPPLY AND DEMAND OF ALPHA

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This paper analyzes the supply and demand for alpha by institutional investors and the money managers who serve them. A large database of products offered by such managers is used to estimate how the demand for such products increases as a function of achieved excess returns and how the ability to produce such excess returns declines with increased AUM (Assets Under Management). Static and dynamic (simulation) analyses are used to explore some implications of these estimates.

1 Introduction

This paper presents a "lean" (parsimonious) model of the supply and demand for alpha by institutional investors and the money managers that serve them. Based in part on similar analyses for mutual funds and hedge funds, we began with a presumption that money managers' ability to supply excess return declines with AUM (assets under management) whereas the demand for their services increases with observed (historic) alpha. Both presumptions were confirmed. In particular, small products supply positive alpha and large products do not.

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Our data is quarterly. By the amount demanded this period (i.e., this quarter) we will mean the inflow of funds rather than total AUM. Inflow may in fact be outflow, so inflow means net inflow which may be negative. Because of differences in the orders of magnitude of the sizes of products, it is most convenient to express inflow as a fraction of beginning-of-period AUM. The demand equation, therefore, has inflow as a fraction of AUM on the left. On the right it has investment alpha, plus a constant and a random term. In addition, it has beginning-of-period AUM, since a product is less likely to have a large percent inflow if it is a large product than if it is a small product.

Of course the alpha in the demand equation cannot be "true, current" alpha, since that is not observable. We use past alpha, measured as outperformance during the preceding three years. We use three years because money managers are usually expected to present performance for the preceding one, three and five years. Since we sought to build a lean model, we chose just one of these. Our hypothesis is that three-year performance is the most important, and is a sufficient one of the three measures.

Outperformance is defined to be portfolio return less benchmark return. As in Sharpe (1992) the benchmark for a product is not based on the name of the product or what a money manager says is its objective. Rather it is a weighted average of certain asset class returns, with weights equal to the regression coefficients of past product performance against past asset class performance. For us "alpha" is the geometric mean of the product's performance minus the same for its benchmark's performance.

The supply equation has the current year's outperformance (product return minus benchmark return) on the left and AUM on the right. As expected large products have more difficulty outperforming their benchmark than smaller products.

The lean model we present here is the result of a process which began with a more complicated model, including lagged endogenous variables, that required simultaneous equation methods to estimate. Successive applications of Occam's Razor peeled away whatever was found to be statistically or economically insignificant. The result was a model which was not only leaner, but one to which single equation methods could be applied, separately, to the supply and to the demand equations. This allowed us to compare the estimation results of OLS with those of "robust regression" procedures not available for simultaneous equation estimation.

Over and above the fact that a quite lean model worked as well as more complicated ones we tried, and the fact that the fit coefficients have the right signs, the following two empirical results seem to us to be noteworthy: the ability to supply alpha declines with increasing AUM, as anticipated, but alpha *is* positive for sufficiently small AUM (not certain *a priori*) and it crosses from positive to negative at product AUM equal to $300 million, a relatively low size for an institutional product.

The second surprise for the authors was the *extent* to which the success of a product was due to luck. We knew, of course, that each equation has a random term, which we fit, using alternate assumptions as reported below. But it was not until we examined simulation outputs that we realized the extent to which a product can rapidly have great success or failure depending on the luck of the draw on the supply and demand sides.

The remainder of this paper is organized as follows: Section 2 acknowledges prior work on the supply of and/or demand for alpha; Section 3 presents our formal model; Section 4 describes the data used to fit the parameters of the model; Section 5 describes the methodology used to make estimates; Section 6 presents the regression results and some static analysis implications; Section 7 explores the dynamic aspects of the model using simulation analysis; Section 8 pulls together "the story" told by the analyses of the preceding sections.

2 Literature review

Our study simultaneously analyzes the supply of alpha by institutional money managers and the demand for manager's services (asset flow) by institutional investors. The majority of prior studies focus on mutual and hedge funds. We however focus on institutional money managers. Perold and Salomon (1991) model theoretical trade costs to argue that as assets under management increase, returns should decrease. They suggest that the "right amount of assets under management" are "surprisingly small" given trade costs. Christopherson *et al*. (2002) and Allen (2007) find empirically a negative relationship between product AUM and excess return across a wide variety of asset classes in the institutional investor universe. Many mutual fund studies study alpha generation given growing fund size, examples including Indro *et al*. (1999), who state that returns become negative when a mutual fund exceeds its optimal fund size, and Grinblatt and Titman (1989), who point out that superior performance may in fact exist in certain styles and with small assets. Our research is also consistent with that of Chen *et al*. (2004) who find that lagged assets are negatively related to performance.

In the hedge fund arena, papers focusing on the supply of alpha as a function of AUM include Fung *et al*. (2006), who find that capital inflows adversely affect ability to produce alpha in the future, while Gregoriou and Fabrice (2002) on the other hand suggest that the size of a hedge fund or a fund-of-hedge-funds has no impact on its performance. Their study deals with returns of hedge funds and funds-of-hedge-funds from 1994 to 1999. Liang (1999) also writes that the average hedge fund returns are related positively to the size of the fund. Boyson (2008) studies hedge funds as well and finds that performance persistence is strongest among small, young funds.

In a study involving international markets Keim *et al*. (2000) show a significant negative relationship in returns and portfolio size in many countries, including eight European countries. An Australian equity funds study in the institutional sphere, by Martin and Gallagher (2005), finds no statistically significant performance differences (net of expenses) between funds on the basis of portfolio size. However, the main finding in the majority of these papers involving the ability to supply alpha as AUM grows, is that as assets under management grow, alpha diminishes.

The other side of our topic relates to the demand function, namely, the impact that performance has on the inflow of assets on the demand for manager's services. Again much of the literature is in the mutual fund setting examples including Gruber (1996), who finds that the flow of new money into the best performing funds is much larger than the flow of money out of the poorer performing funds. Hendricks *et al*. (1993) state that, directly or indirectly, investors in mutual funds are willing to act on such information of relative performance. Chevalier and Ellison (1997) also discuss the relationship between the inflow of assets and returns in a mutual fund setting. Ippolito (1992), Sirri and Tufano (1998), Karceski (2002), and Lynch and Musto (2003) all discuss how inflows follow good performance in a mutual fund setting.

In the hedge fund arena Wang and Zheng (2008) indicate that hedge fund investors as a group chase past aggregate performance. Baquero and Verbeek (2007) find that money inflows are sensitive to past long-run performance and Adams (2007) find performance influences asset flows.

Although we are unaware of previous literature related to using a simultaneous equation model to estimate the supply and demand of alpha and managers services, Snigaroff (2000) discusses the supply and demand of active management, and how over or under demand can affect alpha. Berk and Green (2004) study an equilibrium where the demand and supply intersection is driven towards alpha equal to zero as investors supply funds to managers that result in managers decreasing their ability to supply alpha, which they study empirically in a mutual fund setting. Snigaroff argues that agency issues within plan sponsor organizations can lead to buyers' over-demand which leads to alpha equilibriums of less than zero, consistent with the results shown here. Goyal and Wahal (2008) and Stewart *et al.* (2009) provide recent studies that view institutional flows into and out of asset manager products. Both use comprehensive but different databases to address the question, "Does subsequent performance warrant plan sponsor's decision for product change?" They find that it does not. Although their focus is on individual product performance after a hire or fire decision, the latter finds products with the most inflows performed worse than those with the most outflows. The former note that investment managers who lose a larger fraction of assets have higher post termination returns. Both of these views are consistent with our theme. Finally, in the hedge fund setting Getmansky (2003) models a concave relationship between performance and assets-under-management and relates asset inflows to performance. The paper also describes an optimal asset size that can be obtained.

Overall there is a growing literature on the asset manager size impact on performance and that of performance's influence on the inflow of assets. Our contribution to the subject is (1) our use of a simultaneous equation form to model the interaction of demand (asset flow) with supply (excess return), and (2) with its application to the institutional side of the story.

3 The model

This paper presents a lean model of the supply and demand for alpha by institutional investors and the money managers who serve them. This lean model is the result of a winnowing process in which we deleted, from somewhat more complex models, variables whose coefficients proved to have little statistical or economic significance.[1] The result was the following model of the demand and supply equations, respectively:

$$\log(1 + I_t) = \log(a_\mathrm{d}) + b_\mathrm{d}\log(1 + \alpha_t) + c_\mathrm{d}\log(A_t) + \log(u_{\mathrm{d}t}) \quad (1)$$

$$\log(1 + e_t) = \log(a_\mathrm{s}) + b_\mathrm{s}\log(A_t) + \log(u_{\mathrm{s}t}) \quad (2)$$

where

r_i : is the return on the product during quarter t;

BM_t : is the return on the benchmark during quarter t;

e_t : is the excess return which the manager achieves during quarter t, namely, $r_t - BM_t$;

A_t : is AUM at the beginning of quarter t;

α_t : is $\sqrt[3]{\prod_{i=1}^{12} (1 + r_{t-i})} - \sqrt[3]{\prod_{i=1}^{12} (1 + BM_{t-i})}$; and

I_t : is the inflow of funds during quarter t, net of return and as a fraction of beginning-of-period AUM, namely, $I_t = \frac{A_{t+1}}{A_t(1+r_t)} - 1$.

The random series $u_{\mathrm{d}t}$ and $u_{\mathrm{s}t}$ are i.i.d. In the simulation analysis of Section 7 they are assumed to be lognormal. Equations (1) and (2) imply

$$1 + I_t = a_\mathrm{d}(1 + \alpha_t)^{b_\mathrm{d}}(A_t)^{c_\mathrm{d}}u_{\mathrm{d}t} \quad (3)$$

$$1 + e_t = a_\mathrm{s}(A_t)^{b_\mathrm{s}}u_{\mathrm{s}t} \quad (4)$$

The logarithms in Equations (1) and (2) can be to any base. The A_t series satisfies

$$A_{t+1} = A_t(1 + r_t)(1 + I_t). \quad (5)$$

In particular, a product can grow in size if its return r_t is positive, even if the product experiences an outflow.

4 Data

To fit the coefficients of Equations (1) and (2) we used the eVestment Alliance U.S. Equity Universe database of vendors to institutional investors. Specifically, we consider the performance and the demand for various "products" offered by such vendors. A product is a particular investment strategy managed by a team of investment professionals. Within a particular strategy or product there may be several different vehicles, e.g., a separate account composite, institutional mutual fund, comingled ERISA fund, or comingled Non-ERISA fund, etc. For performance we use the separate account composite vehicle corresponding to each product as the returns for the product. Our product assets under management variable is the total AUM associated with the product or the aggregate of all of the vehicle's AUM associated with a product.

For example, Pyramis Global Advisors Large Cap Value is an example of a product as is Pyramis Global Advisors Large Cap Core; however, the aggregation of these two products is not a product. Within the Pyramis Large Cap Value product they list two different vehicles: a comingled ERISA fund and a separate account composite. We use the separate account performance (managers rarely reported other vehicles) to represent the Pyramis Large Cap Value product performance. For the product AUM, we use the total assets under management contained in both vehicles but do not include Pyramis Global Advisors Large Cap Core.

Within the *eA All US Equity* major class we included products that fall into one of the following of its mutually exclusive subclasses:

eA LargeCap Equity,
eA MidCap Equity,
eA SmallCap Equity,
eA AllCap Equity,
eA MicroCap Equity,
eA SmallMidCap Equity,

each of which contains value, growth, and core styles. In our regression analyses, "returns" are net of fee.

For the present discussion we define an "observation" to be a combination of a product and a quarter. In the first instance we selected observations, during the period Q2/1998–Q3/2008, which met the following criteria:

– The observed product reported returns for 13 consecutive quarters (i.e., the "current" quarter and the 12 preceding).
– The observed product reported AUM for the current and preceding quarter.

These specifications were determined by the requirements of the regression analysis.

We are only considering products with at least \$25 million in AUM. This cutoff value was chosen on the basis of outlier analysis and lead to the deletion of observations which reduced the sample size from 42,283 observations to 38,521. \$25 million is the amount of firm AUM the SEC requires before requiring a firm to be a registered investment advisor.

The computation of the alpha of a product requires that we assign a benchmark to each product. Rather than use the product's stated benchmark we regressed the preceding 12 quarters of return against four benchmark series for the same period. The benchmark for the product at that time was assumed to be the weighted average of the four possible benchmarks, weighted by these regression coefficients. This was used in computing the past alpha for the product that quarter (ex ante) and in computing the excess return (ex post) in the forthcoming quarter. The benchmark return series used were four Russell indices: 1000 Growth, 1000 Value, 2000 Growth, and 2000 Value.

The columns of Table 1 show, respectively: the series (i.e., the Russell benchmarks used, or the performance of all managers in the sample); the periods covered; and their respective arithmetic means, geometric means, and standard deviations of quarterly returns for the sample period.

Table 1 Quarterly return series.

Time period 4/1998–9/2008			
Series	Arith. mean	Geo. mean	Std. dev
Russell 1000 Growth	0.0058	0.0002	0.1069
Russell 1000 Value	0.0130	0.0101	0.0774
Russell 2000 Growth	0.0125	0.0034	0.1365
Russell 2000 Value	0.0218	0.0176	0.0931
Sample Products	0.0123	0.0084	0.0876

This table shows the quarterly return statistics of the benchmarks and of products used in our sample.

5 Parameter estimation procedure

The data described in the preceding section was used to estimate the parameters of Equations (1) and (2). While the two equations interact, as illustrated in our simulation runs, they do not need to be estimated simultaneously. Specifically, the alpha variable in Equation (1) depends only on *previous* excess returns from Equation (2), the excess return term in Equation (2) depends only on the *beginning-of-period* AUM, and the error terms of the two equations are independent of each other as well as independent of the right-hand side variables in both equations. Consequently, ordinary least squares (OLS) provides unbiased estimates for the parameters of each equation, estimated separately.

However, we believe that some form of robust estimation procedure will provide more "representative" results than will OLS. This is not only due to possible data errors. For example, if our objective were to estimate the "typical" wealth of people employed in Redmond, Washington, then a robust estimator like the median would not be affected by Bill Gates' wealth, whereas mean wealth would be influenced substantially. Similarly, it is the median housing price rather than the mean housing price that is reported as representative. The median versus the mean is a special case of robust versus OLS regression, namely, the case in which there are no independent variables on the right-hand side.

MatLab supplies nine methods for fitting the coefficients of linear equations. One of these is OLS; the other eight are various methods of robust regression. We fit coefficients using all the nine methods. In the case of every coefficient, the OLS estimate was an outlier as compared to the other eight and, as it turned out, the Cauchy method of weighting observations provided the median estimate of each coefficient. The Cauchy estimates, used in our simulations are listed in Table 2, along with their t-stats (all significant at the 1% level)

Table 2 Cauchy method robust estimates.

	Demand equation			Supply equation	
	Constant	$\ln(1+\alpha)$	ln (AUM)	Constant	ln (AUM)
Estimate	−0.00840	0.46591	−0.00213	−0.00083	−0.00068
t-stat	−21.97	61.05	−9.42	−5.59	−7.56
Std. err est.	0.00038	0.00763	0.00023	0.00015	0.00009

The authors estimated the parameters of the supply and demand equations using OLS and eight robust methods. The Cauchy method turned out to be the median estimate for every parameter. This table lists the regression results of the Cauchy method. Specifically, regression coefficients along with their corresponding t-statistics and standard error estimates are listed for both the demand and supply equations given in Equations (6) and (7).

and their standard errors of estimation. The results for all methods are presented in Wroblewski *et al.* (2009). DuMouchel and O'Brien (1989), Holland and Welsch (1977), Huber (1981) and Street *et al.* (1988) are the references to robust regression cited by MatLab.

In addition to regression coefficients, our simulations need estimates of the standard deviations of the random terms of Equations (1) and (2). The observed deviations from the regression fit in Equations (1) and (2) are due to two sources. Specifically, we must distinguish between the *between-product* variability and the *within-product* variability. As is well known, the variance of each residual term is the sum of its within-product variance and its between-product variance. For the purpose of the simulation we need to use the "typical" *within-product* variance.

To estimate this we sorted the residuals from the (Cauchy robust) regression supply residuals (demand residuals) into "buckets," one bucket per product, and computed the within-bucket variance for each product, corrected for degrees-of-freedom. The results of these calculations are shown in Table 3A for demand residuals and Table 3B for supply residuals.

The first line of each table shows the breakdown of total variance between within-product and between-product variance, not corrected for degrees-of-freedom. (Note that, for both the supply and demand residuals, the within-product variances are much larger than the between-product variances.) The second lines show the respective standard deviations. It is, of course, the within- and between-product *variances*—rather than their standard deviations—which sum to that of the total.

The next two lines of each table summarize the results of correcting the within-product variances for degrees-of-freedom. The first of these lines shows the mean of the corrected variances, the second shows the median. The following two lines show corresponding standard deviations, namely, the square roots of the mean of the

Table 3 Residual decomposition.

	Within products	Between products	Total
A. Demand residuals			
Variance (uncorrected)	0.05947	0.00883	0.06830
Standard deviation	0.24386	0.09397	0.26134
Mean corrected variance	0.09640		
Median corrected variance	0.01377		
Square root of mean variance	0.31048		
Square root of median variance	0.11732		
B. Supply residuals			
Variance (uncorrected)	0.00120	0.00009	0.00130
Standard deviations	0.03468	0.00963	0.03599
Mean corrected variance	0.00139		
Median corrected variance	0.00075		
Square root of mean variance	0.03728		
Square root of median variance	0.02731		

This table shows the decomposition of the total residual variance for the demand and supply equations into a within-product variance and a between-product variance. The simulation uses the within-product variance. We also correct for degrees of freedom within each product and display the mean and median of this corrected variance, across all products, as well as the associated standard deviation.

corrected variances and the median of the corrected variances.

Since there is a substantial difference between the mean and the median of the within-product variances, we ran our simulations with a range of estimates, to observe the sensitivity of the simulation results to these parameters.

6 Regression results

The robust Cauchy fit coefficients for Equations (1) and (2) are

$$\ln(1 + I_t) = -0.0084 + 0.4659 \times \ln(1 + \alpha_t) - 0.0021 \times \ln(A_t) \tag{6}$$

$$\ln(1 + e_t) = -0.00083 - 0.00068 \ln(A_t). \tag{7}$$

The coefficients reflect the fact that I_t and e_t are quarterly observations. A_t, of course, is as of a point in time, namely, the beginning of quarter t.

These results are illustrated in Table 4. The first column shows various AUM in billions of dollars. The second column shows the expected quarterly excess return of the product according to Equation (7). In particular, expected excess return is positive at \$0.1 billion and negative at \$1.0 billion. Setting $e = 0$ in Equation (7) we see that, according to this equation, excess return crosses from positive to negative at a product size of \$295 million. In other words, below an AUM of about \$300,000,000, average products produce excess return; above it, they do not. At a product size of \$10 billion, expected underperformance is 24 basis points per quarter: almost 1% per annum.

Table 4 Expected inflow and excess return as a function of AUM.

AUM	Supply equation excess return (percent per quarter)	Demand equation inflow (percent per quarter)
0.01	0.23	0.58
0.10	0.08	−0.21
1.00	−0.08	−0.99
10.00	−0.24	−1.76
100.00	−0.40	−2.53

This table illustrates the results of the regression analyses for the supply and demand equations. The first column lists various levels of AUM. The second column shows the estimated excess return per quarter that the product would achieve with that level of AUM. The third column shows the associated inflow or outflow per quarter, assuming the excess return in the second column (annualized) is the observed alpha.

The third column of Table 4 shows the estimated inflow or outflow at the various AUM. If there were no random terms in either Equations (1) and (2), nor in the benchmark return, and A_t were at the deterministic equilibrium value implied by Equations (5)–(7), then the same value of A_t would repeat each quarter, resulting in the same excess return each quarter. This repeated excess return, annualized, would then be the α_t in Equation (6). This alpha, together with the A_t from column 1, was used in Equation (6) to obtain the resulting inflow or outflow reported in column 3. For example, the third line of the table shows that a product with an AUM of 1 (billion dollars) has an (expected) excess return of negative eight basis points per quarter which, at that level of AUM, leads to an (expected) outflow of almost one percent per quarter.

But a negative inflow (i.e., an outflow) does not necessarily mean that AUM falls. To obtain the change in AUM one must sum the product's inflow (or outflow, if negative) *plus* its excess return, *plus* the return on the benchmark, in accord with Equation (5) (neglecting the $r_t I_t$ term, for the moment and using the fact that return r_t is the sum of benchmark return and excess return). Thus if the benchmark returned 2% per quarter, a 10 billion dollar product would approximately hold its own. At 1.94% per quarter, which compounds to 8% per annum, an exact calculation (including the $r_t I_t$ term) shows static equilibrium at \$8.02 billion (see Wroblewski *et al.*, 2009). For a benchmark return of 7% per annum (1.71% per quarter), as assumed in the simulation reported in the next section, the static equilibrium is only \$4.53 billion.

7 Simulation analysis

The previous section described model equilibrium if the supply and demand equations had no random terms and the benchmark were constant over time. We used simulation analysis to analyze system dynamics including its random terms. Specifically, we ran simulation analyses using the coefficients in Equations (6) and (7) with varying estimates of σ_d and σ_S, and with varying means and standard deviations of the benchmark. The results varied numerically, depending on the inputs, but the qualitative characteristics were the same for all cases considered. In this section, we first present one of these cases, and then discuss the sense in which the results of the other cases were qualitatively the same. See Wroblewski *et al.* (2009) for details of the other cases.

In the case presented here we assume that the benchmark has a geometric mean of 7% per annum[2] and a standard deviation of 20% per annum; and that the demand and supply standard deviations are $\sigma_d = 0.20$ and $\sigma_s = 0.025$ per quarter. The σ_d^2 used here is between the mean and the median variance of the historic within-product variances, corrected for degrees of freedom. The σ_s^2 is roughly the median corrected within-product variance of the supply equation.

Figures 1–4 show the results of 500 simulation runs. In each run a simulated product (already in existence) starts the run with a \$1 billion AUM and 12 preceding quarters with zero excess return. From this common starting point, 500 alternate histories of "the product" evolve according

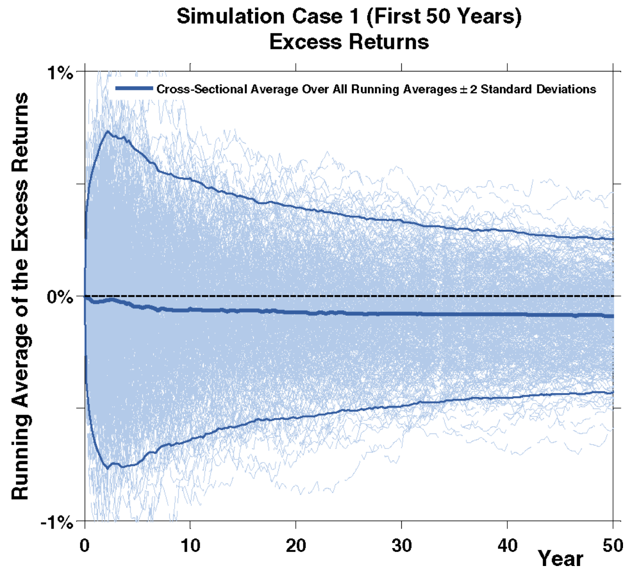

Figure 1 For one of many simulated cases, this figure displays the average-to-date quarterly excess return, for 50 years worth of quarters, for 500 runs (replications) of the case. The light blue lines show this average for each run. The dark blue lines show the cross-sectional average, and that plus or minus twice the cross-sectional standard deviation. After 50 years the average of the average-excess-return-to-date was a negative nine basis points per quarter.

to Equations (5)–(7) from 500 series of i.i.d. lognormal draws of u_d, u_s and the benchmark.

For each of the 500 random replications, Figures 1 and 2 plot the cumulative average excess return:

$$\bar{e}_t = \frac{1}{t}\sum_{i=1}^{t} e_i, \quad \text{for } t = 1, \ldots, T \tag{8}$$

for T equal to 200 (50 years worth of quarters) in Figure 1, and 2000 (500 years worth of quarters) in Figure 2. (We ran the simulation for 500 years to see if the simulations converged to equilibrium, and how long it took to get there.) If e_i were i.i.d., the strong law of large numbers says that $\bar{e}$ would converge almost surely to the expected value of e_i. The result for each individual run is a light blue line. The solid dark blue lines in the two figures

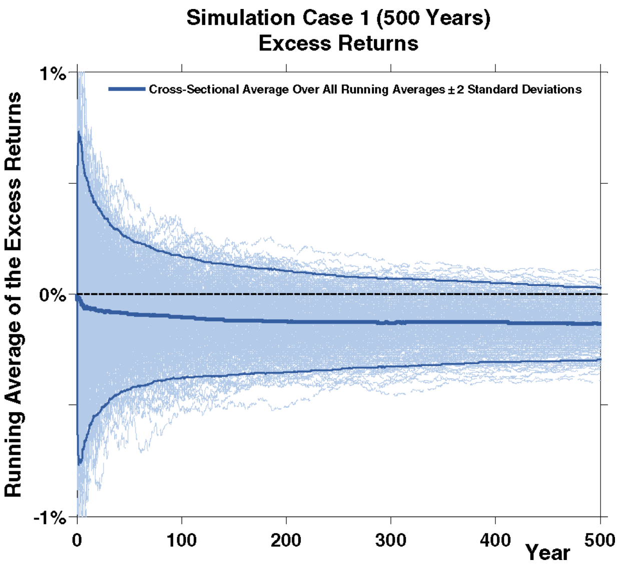

Figure 2 This figure is the same as Figure 1 except that average-to-date excess quarterly returns are shown for 500 years. By this time the average of the average-to-date excess returns is a negative 13 basis points per quarter. The purpose of such long simulated to runs is to examine the convergence properties of the model. One clear conclusion is that, by human standards, the model takes a very long time to converge.

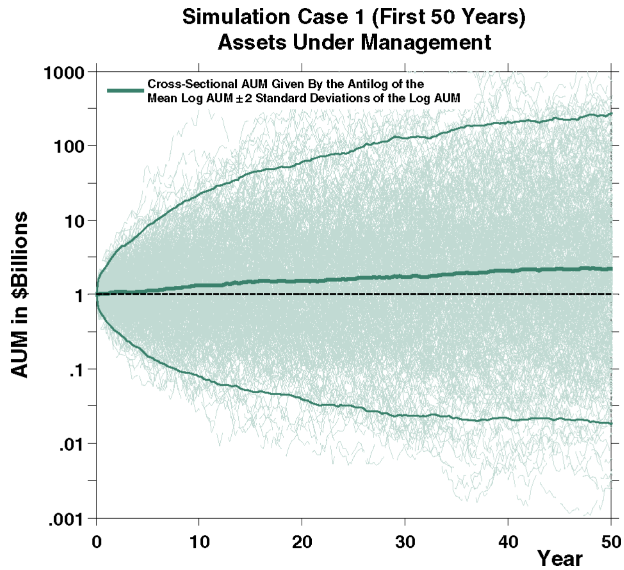

Figure 3 This figure displays the Assets Under Management (AUM) for the same 500 simulated products as in Figures 1 and 2. In each case initially AUM = 1.0 billion dollars. Note that after only ten years, replications of the identical products—except for the luck of the draw—range from of a mean-minus-two-sigma of about 0.1 billion dollars to a mean-plus-two-sigma of 10 billion dollars. By the 50th year the mean-plus-and-minus two sigma range is from about 0.01 billion to about 100 billion.

show the *cross-sectional* mean, and the mean plus and minus two standard deviations, averaged at time t over the 500 replications. The broken horizontal line is the $\bar{e} = 0$ line. By the end of the 50th year $\bar{e}$ was roughly a negative nine basis points per quarter, and by year 500 it was a negative 13 basis points per quarter.

Figures 3 and 4 show A_t for $t = 1, \ldots, T$, where $T = 200$ and 2000 quarters, respectively. Because of large differences in the product AUM, a semi-log plot is used. We plot log (AUM) rather than $\frac{1}{t}\sum_{i=1}^{t} \log(A_i)$, since $\log(A_t)$ is itself already a sum, as seen by taking the logs of both sides of Equation (5).

The dark green lines in Figures 3 and 4 show the AUM's which are the antilogs of the cross-sectional mean log AUM and the mean log AUM $\pm 2\sigma$ (of the log AUM) computed over the 500 replications represented by the light green lines. After 50 years the cross-sectional geometric mean (i.e., the antilog of the arithmetic mean of the log AUMs) is \$2.22 billion; by the end of 500 years it is \$4.33 billion, as compared to \$4.53 billion for the static calculation. As of year 50, the cross-sectional standard deviations are still increasing. By year 500 (in fact, by roughly year 150) the cross-sectional standard deviation has converged to a steady state. Figure 3 shows that after about 10 years, for example, the plus and minus two standard deviation lines incorporate a large range of AUM, from under \$100 million to over 10 billion dollars, depending on the luck of the draws with respect to the u_{s} and u_{d}. This range keeps expanding as just noted.[3]

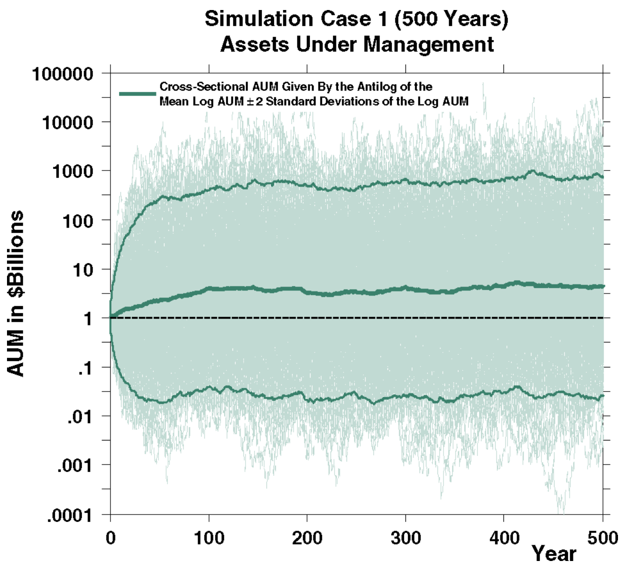

Figure 4 As in Figure 3, this figure shows simulated AUM for 500 cases, but here for 500 years rather than 50. The cross-sectional geometric mean of the AUM converges to $4.33 billion, with AUM close to its ultimate value after about 100 years. As noted on Figure 2, we realize that these are beyond product lifetimes, but we ran the model out for 500 years to see its convergence properties.

In addition to the case presented here, we simulated various other cases: with the geometric mean of the benchmark varying from 6% to 9% per annum, with σ_{d} varying from 0.068 to 0.300 and σ_{s} equal to 0.025 or 0.035. In the cases tried, the results were qualitatively the same as the case reported in Figures 1–4, in the sense that,

- Average AUM increased during the first 50 years (by about two-fold to about five-fold depending on case parameters) and continued to increase over the next 450 years (by roughly another factor of two).
- Average excess return was negative at the 50 year mark, and even more so over 500 years.
- The model converged, but very slowly in terms of human (and product) time scales.

The most striking feature of all cases was the great dispersion of results both in the short and long run. If we examine the first few years of the product's existence—even though each case represents a randomly drawn path of essentially the same "representative" product—there are huge differences in AUM, and consequent differences in expected returns, depending on the luck of the draw.

8 The story

Our data and analysis tell the following story concerning products offered by vendors to institutional investors. We considered products at points in time at which they had been in existence for at least 13 quarters and had at least $25 million under management. Among such products we find that—on average—the smaller products outperform their benchmarks and the larger products do not. The break-even level is about $300 million. An order-of-magnitude increase in AUM leads to a decrease in excess return of about 16 basis points per quarter (64 basis points per annum).

Products can grow in size despite negative excess returns as long as, on the demand side, the outflow of funds is less than the internal rate of growth of the product. The level of AUM at which outflow equals internal growth, so that the size of product is stationary, depends on benchmark growth, since return is benchmark return plus excess return. Thus the static analysis did well in telling us where the dynamic analysis was headed but, of course, did not tell us how long it would take to get there. If we use the fit coefficients and a benchmark growth rate of 7% per annum, the static calculation, assuming $u_{\mathrm{s}} = u_{\mathrm{d}} = 0$ and no variance of the benchmark, shows an equilibrium of $4.53 billion, whereas the dynamic simulations show an AUM equilibrium of $4.33 billion. In particular, a lower equity premium means a smaller equilibrium asset manager size.

The supply and demand equations have substantial quarter-to-quarter variability (on the demand and on the supply side) for a given product. This quarter-to-quarter variability is much larger than the variability between products. As a result, in the short run equally good products can experience substantial differences in performance, leading to great differences in inflow and consequent orders of magnitude differences in AUM—depending on the luck of the draw. In the long run, the cross-sectional distribution of the cumulative average of excess returns and the cross-sectional geometric mean of AUM converges, but very slowly in terms of human and product lifetimes.

Notes

[1] The following variables were considered in models leading up to the lean model presented in the text.

Demand only:

1. Current AUM.

Supply only:

1. Number of Holdings (lagged one year).

Demand and supply:

1. Change in real GDP.
2. Weighted average of the number of portfolio managers (PM) and the number of analysts.
3. Number of products in firm.
4. Lagged inflow.
5. Lagged alpha (3yr alpha-lagged one year).

In addition to low economic or statistical significance, variables were deleted because reliable quarterly data was not available. Often quarterly data would be reported but, suspiciously, would be the same each quarter of the year. Inclusion of these variables would have required us to use annual data and a shorter time period. This would have cut our sample size by more than a factor of four. In particular, we would have had a maximum of eight observations per product line rather than the current maximum of 42.

[2] Mindful of arguments such as those of Arnott and Bernstein (2002) that rate of return seen over the past many decades should not be expected in the future, since there was a great increase in valuations over the period.

[3] Since the AUM converges in nominal terms, over the long run it must approach zero in real (constant dollar) terms as long as the inflation rate exceeds zero. This is not a necessary implication of the model in Equations (1) and (2), but implications of the statistical fit reported in Table 2. For a model consistent with Table 2, but without real AUM asymptotically zero, we could drop the assumption implicit in Figures 1–4, that all replications of "the product" remain "forever," and no new ones come into existence. We could, instead, assume that those products whose real AUM falls below some lower level disappear and (on average) an equal number of new ones keep appearing with a real $1 billion starting value.

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