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A Network Value Theory of a Market, and Puzzles

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By considering the stock market as a network that impounds liquidity and information production, the authors were able to study its influence on aggregate stock value and value from dividends. Market participants and practitioners impart value through the network of activity they form. The authors offer a network value model that can price this value and help solve such financial economic puzzles as the equity premium, stocks' inverse inflation relationship, and lack of news.

The information and liquidity infrastructure of the public stock market is a complex and valuable network that has been in continuous existence for several hundred years. Ironically, it is assigned no value in frictionless market and complete information models. Indeed, in the popular press, investment valuation practitioners are even compared to "monkeys" and "dartboards." The premise of our study is that the infrastructure built by market participants and practitioners not only is valuable and material but also can be measured. With that in mind, we built a model to measure the infrastructure, proceeded to show that the model fits actual prices well, and then used it to help solve the equity premium, inverse inflation, and market-making-its-own-news puzzles. We developed our model by adding to the dividend discount model (DDM) of Williams (1938) and Gordon and Shapiro (1956).

Shiller (1981) relied explicitly on the DDM to build an *efficient market model*[1] and, by approximating the index level, P_t, with

$$P_t^* = \sum_{k=0}^{\infty} \frac{D_{t+k}}{\left(1+r\right)^{k+1}}, \tag{1}$$

where D_t are the dividends and r is the discount rate, suggested that variability in the numerator is too small to justify the volatility in stock prices. Mehra and Prescott (1985) argued that a consumption-based utility model justifies a much smaller equity premium than is usually observed. That the compound growth rate of the index level, P_t, over time has been so large relative to that of both Equation 1 and bonds presents a conundrum: the equity premium puzzle. Fama and Schwert (1977) and others have pointed out that stocks have an inverse inflation relationship. When inflation occurs, companies should be able to raise their prices. Their customers should receive (or expect) higher nominal wages and be willing to pay a company's higher prices; hence, company revenues should rise, leading to a rise in nominal earnings and dividends, according to Fisher (1930). The difficulty in understanding why this mechanism should not be enough to offset a rise in the discount rate is often called the inverse inflation puzzle. Finally, the market frequently experiences very large movements that seem unrelated to any component of Equation 1—that is, the market seems to make its own news.

These are admittedly simplistic renderings of complex puzzles, each of which has an active and extensive literature replete with proposed solutions. A simple approach to solving these puzzles, which might otherwise seem unrelated, is to assume that the DDM is incomplete. The incompleteness of the DDM (and of asset pricing models in general) is hard to see for two reasons. First, there is no promised or expected cash flow (beyond dividends and buybacks) from the companies themselves. Second, ignoring Hayek's information argument (1937), asset pricing theory starts with the assumptions that markets are complete, all investors have all information, and securities are perfectly liquid. By not making these simplifying assumptions, we were able to study what happens to asset prices vis-à-vis the changing level of market "completeness."

We assumed that all the activities of market participants form a "network." We could then abstract the value of this network from the level of its network traffic—that is, its volume of trading on the NYSE, the leading U.S. stock market for the 108-year span that we studied (for econometric reasons discussed later in the article, we used turnover).[2] This network has value that is similar to the network value of the worldwide internet. The activities of market participants who build this

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network produce value: thus, our "network value theory." Although modeling the value contributed by the network components of liquidity provision, information production, and even participants' confidence in the functioning of the network is straightforward,[3] we did not do so in our study (in the network literature, this approach is deemed a "macro-network" argument). We abstracted from volume the changes over time in our network value (NV) function and decomposed stocks' value from dividends alone and the NV component to study their asset pricing effects.

Network Value Model

Our network value model contains the following components:

- P_t = the beginning of time t index level
- D_t = the dividends paid at the end of time t
- TO_t = the share turnover over the period $t-1$ and ending at time t
- $\mathcal{N}$ = a general time series used as a proxy for the production of the network
- R_t = the total return over the period t
- $\mathcal{I}_t$ = the information set available to investors at time t
- $E(\cdot)$ = the expectation operator

The simple efficient market model begins with the definition of total return, R_t—namely, that the return is given by a capital gain contribution and a dividend contribution:

$$R_t = \frac{\left(P_{t+1} - P_t + D_t\right)}{P_t}. \tag{2}$$

By assuming that the conditional expectation of R_t with respect to the information available at time t is constant, one obtains the following formula:[4]

$$\frac{P_t}{D_{t-1}} = \sum_{k=0}^{\infty} \frac{E\left[(D_{t+k} / D_{t-1})\middle|\mathcal{I}_t\right]}{(1+r)^{k+1}}. \tag{3}$$

Equation 3 is the classical DDM, with the discount rate equal to r after dividing through by D_{t-1}. It models the price with a sum of discounted expected future dividends. Although the idea of discounting future cash flows is both intuitive and useful, we believe that it misses the value given to stocks by the market's functioning as a network. The market's existence raises security prices, which, in turn, lowers one-step-ahead future returns. We account for this decrease in future returns, which arises from an increase in current prices, by adding a factor to our return equation. This addition also allows us to capture and quantify the additional network value. Proxies for the production of the network, $\mathcal{N}$, could include the number of investors, brokerage revenues, the price of exchange seats, turnover, and other measures. We do not use a nontransformed $\mathcal{N}$ alone for this factor because we believe that the value added by the market network is not simply linearly related to $\mathcal{N}$ but, rather, includes the notion of increasing followed by decreasing returns to scale.

Modeling network growth as an S-shaped function is common practice in the network literature (for a discussion and summary of functional forms, see Swann 2002). Until the market reaches "critical mass"—the idea that a network needs to grow to sufficient size in order to overcome what Rohlfs (1974) called the "start-up problem" (see also Marris 1964; Artle and Averous 1973)—the rate of change of the network function slowly increases. After some level of production, our function has a negative second derivative because the rate of change decreases after the market reaches a fully functioning level. We shift our function to pass through the origin in order to signify that if a market does not exist, the network adds no value. With respect to financial markets, participants have long observed similar growth patterns for many different markets and instruments. Moreover, S-shaped growth functions are commonly assumed (and demonstrated in virtually all microeconomics texts) for the general growth of products, markets, and even economies. With that in mind, we define function $\theta_{\mathcal{N}}$[5] by using the normal distribution's cumulative distribution function and arrive at the heart of our model:

$$\frac{P_t}{D_{t-1}} = \sum_{k=0}^{\infty} \frac{E\left[\left(D_{t+k}/D_{t-1}\right)\middle|\mathcal{I}_t\right]}{(1+\delta)^{k+1}} + \sum_{k=0}^{\infty} \frac{E\left[\theta_{\mathcal{N}}\left(\mathcal{N}_{t+k}\right)\left(D_{t-1+k}/D_{t-1}\right)\middle|\mathcal{I}_t\right]}{(1+\delta)^{k}}. \tag{4}$$

The constant, δ, represents the discount rate. Unlike in the DDM, however, we allow for the return estimate to change over time on the basis of our NV function, θ (see Appendix A for more details). On the right-hand side of Equation 4, the first term can be viewed as the expected proportional cumulated growth in dividends and the second term as a covariation between the network production function and the proportional cumulated growth in dividends. Thus, we assume that the index level is determined not only by an expected future dividend stream but also by a value from a network.

An important distinction should be made between Equations 3 and 4. The latter differs from the DDM in that it expresses the price not only as a sum of discounted expected future dividends but also with an additional sum of discounted expected future covariation terms involving the network proxy and dividend processes. Although the first term in the network value model (NVM) is very similar to that of the DDM in that it also is a sum of discounted expected future dividends, it is subtly different. Because the second term is positive and both models are expressions of the price, P, the first term in the NVM must be smaller than the corresponding sum in the DDM. This condition is met by using a discount rate, δ, in the NVM that is larger than the discount rate, r, in the DDM.

Approximating the DDM and the NVM. Shiller (1981, 1992) provided a well-known efficient market DDM that defines P_t^* as

$$P_t^* = \sum_{k=0}^{\infty} \frac{D_{t+k}}{(1+r)^{k+1}} \tag{5}$$

so that P^* is the present value of the infinite future dividend stream. Shiller's model also states that the market is efficient and that the true price—in this case, the index level—is

$$P_t = \mathrm{E}\left(P_t^* | \mathcal{I}_t\right) = \sum_{k=0}^{\infty} \frac{\mathrm{E}\left(D_{t+k} | \mathcal{I}_t\right)}{(1+r)^{k+1}}. \tag{6}$$

In the same spirit, our NVM adopts similar notational definitions:

$$P_t^{**} = \sum_{k=0}^{\infty} \frac{D_{t+k}}{(1+\delta)^{k+1}}; \tag{7}$$

$$P_t^{\dagger} = \sum_{k=0}^{\infty} \frac{\theta_{\mathcal{N}}\left(\mathcal{N}_{t+k}\right)\left(D_{t-1+k}\right)}{(1+\delta)^{k}}; \tag{8}$$

$$P_t^{(\dagger,*)} = P_t^{**} + P_t^{\dagger}. \tag{9}$$

We use P^{**} in Equation 7 to differentiate it from Shiller's P^*. Under these definitions, we have

$$P_t = \mathrm{E}\left[P_t^{(\dagger,*)} | \mathcal{I}_t\right] = \mathrm{E}\left(P_t^{**} | \mathcal{I}_t\right) + \mathrm{E}\left(P_t^{\dagger} | \mathcal{I}_t\right). \tag{10}$$

P^{**} and $P^{\dagger}$ are disaggregated components of $P^{(\dagger,*)}$, which approximates the price, P. This decomposition is very useful in understanding the components of stock returns and how they may contribute to the asset pricing puzzles that we discuss later in the article.

Although P^* and $P^{(\dagger,*)}$ are never observable, we may approximate these values *ex post* by truncating the infinite tail. Following Shiller (1981, 1992), we fix a terminal value ($P_T = P_{2009}$) that corresponds to the actual index level in 2009 and use the following recursion to approximate P^* by working backward from terminal time T:

$$P_t^* = \frac{1}{1+r}\left(P_{t+1}^* + D_t\right). \tag{11}$$

Similarly, we may approximate $P^{(\dagger,*)}$ by using the recursion

$$P_t^{(\dagger,*)} = \frac{1}{1+\delta}\left[P_{t+1}^{(\dagger,*)} + D_t\right] + \theta_{\mathcal{N}}\left(\mathcal{N}_t\right)\left(D_{t-1}\right). \tag{12}$$

This approach provides a procedure for using sample data to approximate the price series in two different ways and allows us to compare these two model formulations.

Fitting the Model and Assumptions. Note that we have not built the value of the network from the number of users, the number of connections, the number of groups, the number of traders, and so on. Instead, we have abstracted the value directly from network traffic (i.e., share turnover). Conceptually, this approach is similar to abstracting the value of the internet from the number of "page views" (i.e., the number of user requests to click on a website) or terabytes of volume on the internet over time—standardized by the number of users, IP addresses, connections, or computers. For some networks, measurement issues would make abstracting from the volume of use more problematic. Because of trading costs, aggregate volume as a measure of value for the entire market network likely gives a more accurate measure than suggested by this internet example. Therefore, throughout this article, we compare our approximations of P^* and $P^{(\dagger,*)}$ by using the turnover series, TO, in place of $\mathcal{N}$.[6]

The construction and interpretation of our NVM, which adds to a rational DDM, rely on some key assumptions: (1) The market network has positive value, (2) a turnover function is a useful measurement of this value, and (3) volume is rational. Regarding the last assumption, if markets have "fads," volume surely rises with exuberance and falls with pessimism. In our study, we did not attempt to parse the information and liquidity-based volume from "noise" trades, a simplification required by our dataset; moreover, economic modeling generally starts with investor rationality in the first iteration. With respect to the second assumption, although we could have considered other variables, assuming that the equilibrium is made apparent via the execution of share trades seems reasonable; one might well conceive of changes in equilibrium without volume, but we chose not to. In addition, the available aggregate NYSE turnover data cover more than 100 years. Under the first assumption (that the network always has positive value—its existence does not subtract value from stocks), our functional form posits that the network starts out with a value of zero (no volume), increases slowly at first, then more quickly, and finally "flattens out" as volume increases. Although we considered a model with diminishing value from too much turnover, it delivered weaker results.

Model Results. Following Shiller (1992) in computing P^*, we can use the geometric mean of index-level total returns over the entire sample period, 8.94 percent, as the approximation of r. Using $\mathcal{N} = TO$, we can estimate the coefficients in Equation A6 ($\hat{\delta}$ = 13.35 percent and $\hat{\alpha}$ = 1.66), which allows us to approximate $P^{(\dagger,*)}$. Holding these coefficients constant produces a "fixed" function that does not change over time.

Figure 1 shows a time series of values of this particular NV function, θ; these values are used to construct $P^{(\dagger,*)}$. Relatively high in the early decades, the network value then subsides and ultimately gains strength over time. One useful point to recall, however, is that the value of the NV contribution is not given by the heights depicted in Figure 1 alone but, rather, is a discounted sum of the products of those values and the dividend process as shown in Equation 8.

Figure 2 and **Figure 3** show the P_t^*/D_{t-1} approximation from Shiller's DDM and the $P_t^{(\dagger,*)}/D_{t-1}$ of our NVM compared with the actual P_t/D_{t-1} over various periods.

Depicting a measure of the "closeness" of the two approximations to the actual P/D, **Figure 4** includes a probability density approximation plot for the three series. (See Appendix B for additional technical information describing this plot as well as other measures of the superior fit of our model.)

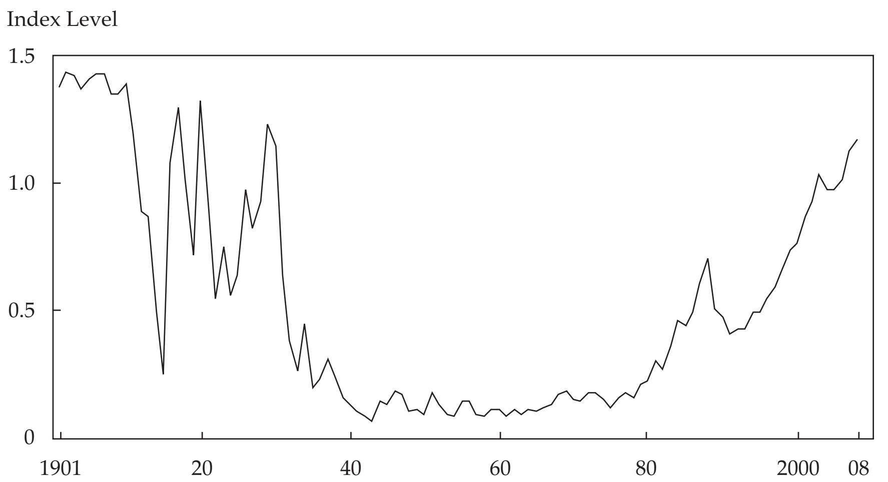
Time-Varying NV Function. We can now compute $P^{(\dagger,*)}$ as we did in Equation 4, with the addition of a time-varying discount rate and a time-varying NV function to account for investors' nonstationary views of the market:

$$P_t = \sum_{k=0}^{\infty} \frac{E\left(D_{t+k} \mid \mathcal{I}_t\right)}{\Pi_{j=0}^{k}\left(1+\delta_{t+j}\right)} + \sum_{k=0}^{\infty} \frac{E\left[\theta_{(\mathcal{N},t+k)}\left(\mathcal{N}_{t+k}\right)\left(D_{t-1+k}\right) \mid \mathcal{I}_t\right]}{\Pi_{j=0}^{k-1}\left(1+\delta_{t+j}\right)}, \tag{13}$$

where $\Pi_{j=0}^{-1}\left(1+\delta_{t+j}\right)=1$. We can then run the same analysis as before, with the addition of a time-varying $\theta_{\mathcal{N}}$ and δ. Again, with $\mathcal{N} = TO$, we can approximate $P^{(\dagger,*)}$ in the time-varying case by using recursive nonlinear least squares. We can also incorporate the additional rule that if at any stage in the recursive least squares we obtain a negative value for the NV function's coefficient, we use the value of the closest past period's coefficient, which has a non-negative value. This choice, which occurred once out of the 99 parameter estimates, may be interpreted as investors' not changing their beliefs unless the new information is relevant.

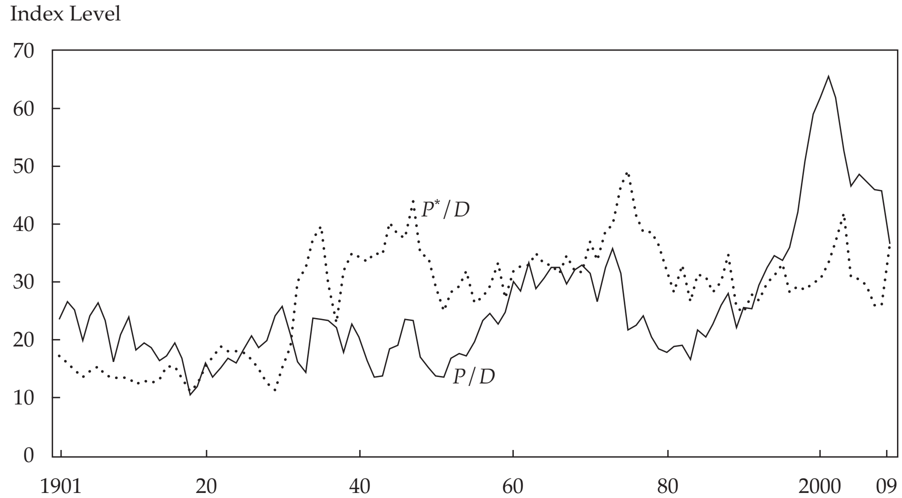
We can also compute the analogous time-varying version of P^* and compare it with the time-varying version of $P^{(\dagger,*)}$. This computation is done as before except that the discount rate varies over time and is defined at time t as the geometric mean

Figure 1. Fixed NV Function, 1901–2008



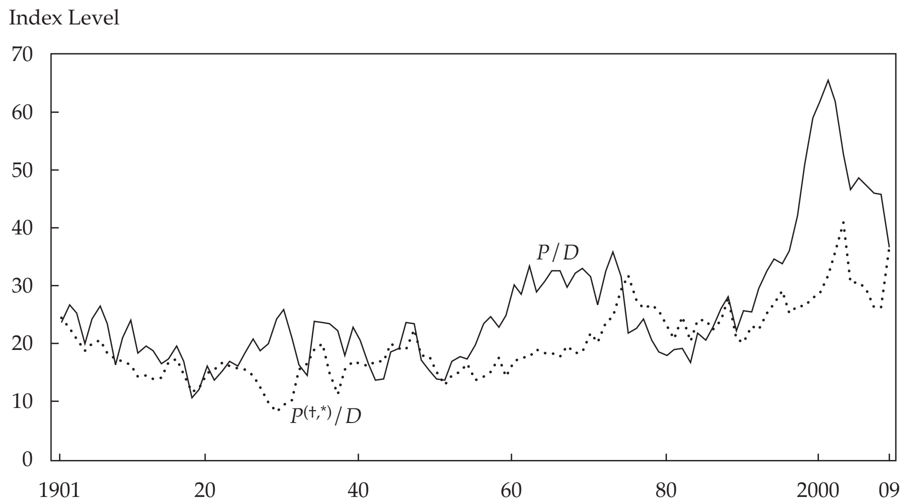
Notes: This figure depicts the values of the NV function used in the construction of $P^{(\dagger,*)}$ over time; each value can be thought of as the relative magnitude of the network value for any given year. Note the low values from the 1930s through the 1970s.

Figure 2. Shiller DDM Approximation of *P*/*D*, 1901–2009



Notes: This figure shows Shiller's DDM approximation, P^*/D, compared with the actual P/D. Note the wide divergence from the actual price-to-dividend ratio over the period (with a few exceptions). The correlation coefficient is 0.30.

Figure 3. DDM with a Fixed NV Function Approximation of *P*/*D*, 1901–2009

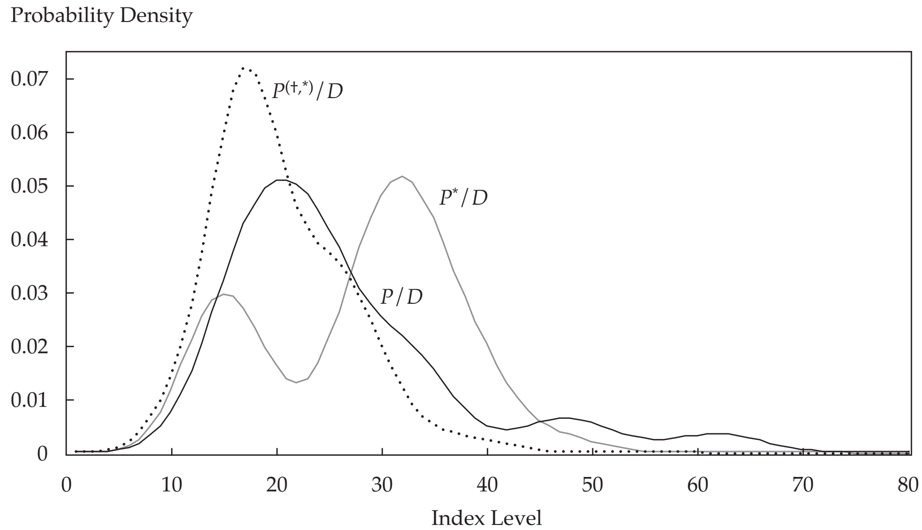


Notes: This figure shows $P^{(\dagger,*)}/D$, the DDM with a fixed NV function approximation, compared with the actual P/D. The addition of an NV function that incorporates stocks' value from the functioning of the market provides a much better fit and is closer to actuality. The correlation coefficient is 0.70. Both the DDM and the NVM "miss" around 2000, which could be considered a bubble period.

of index-level total returns over times from 1 to t. **Figure 5** depicts the time-varying NV function by decade. The network effect follows the previous pattern of strong, subsiding, and then strong again. In the early decades, the network functions themselves (the plotted curves) are more dispersed than they are later on. Two likely causes are that the network effect is itself more volatile, as is the convergence of $\theta_{(\mathcal{N},t)}(\cdot)$ over time to the steady state $\theta_{\mathcal{N}}(\cdot)$ from the fixed NV function of the full sample. In the interest of space, analogs to Figures 2 and 3 for the time-varying NV function (which are very similar) are not presented.

Alternative Variables. The NVM expresses the added value of a functioning market by incorporating an additional term into the DDM framework. In comparing the NV approximation results with those of the DDM, one might naturally ask whether the gains in explanatory power are from

Figure 4. Probability Density Approximation Plots



Notes: This figure plots the probability density function of the actual price-to-dividend ratio as well as the probability density function of both the Shiller DDM and the NVM approximations of the actual price-to-dividend ratio. Using the density plot of the actual price-to-dividend ratio as a benchmark, we can see that the density plot of the NVM approximation is much closer in shape and location to the benchmark than is the density plot associated with the Shiller model. The density plots not only describe the structure associated with the random variables but also may be used to compute probability statements relating to them.

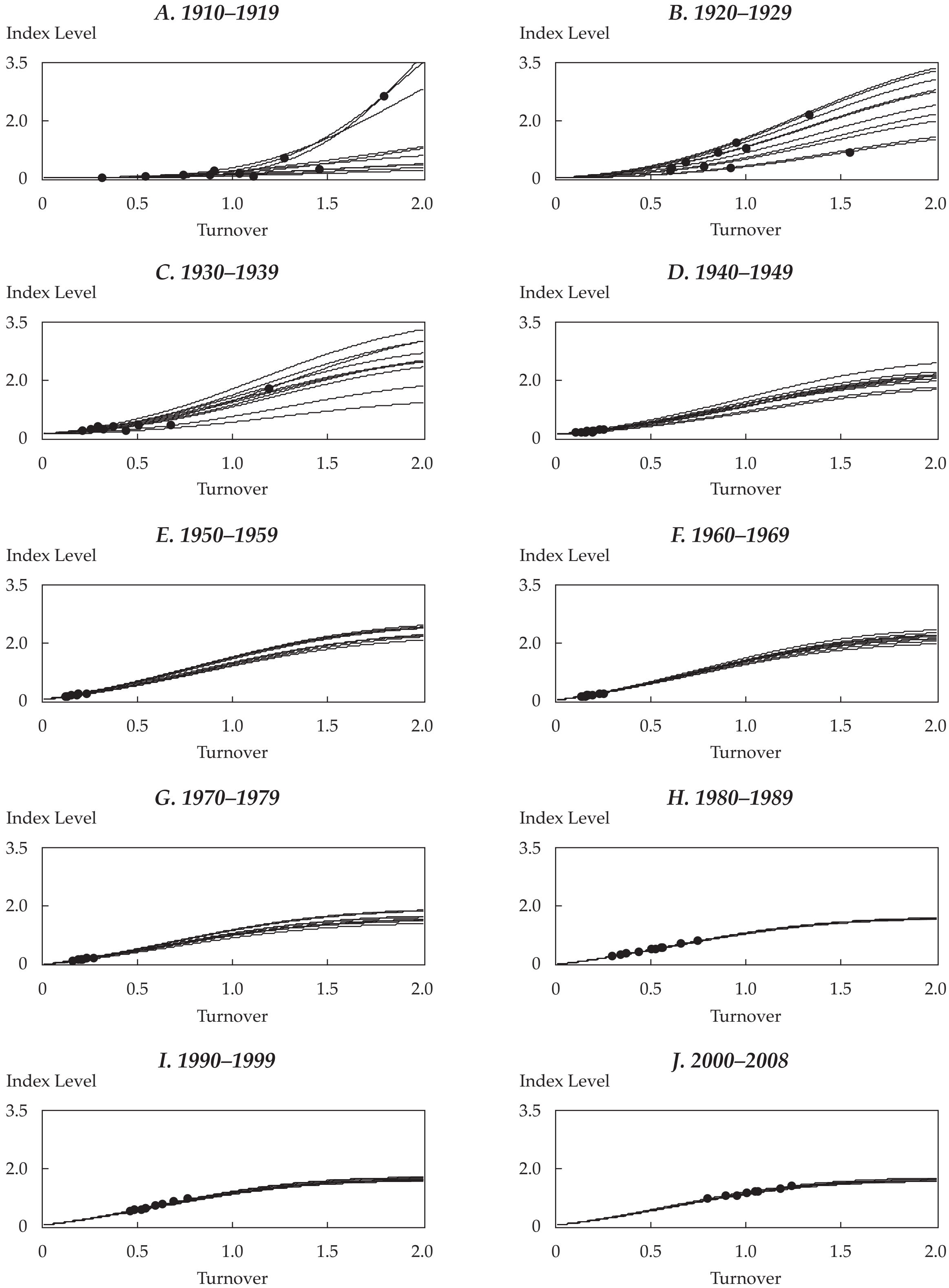
the addition of our NV component or simply from the addition of any arbitrary term. Thus, we also considered the effect of the addition of variables based on GDP, term spread (defined as the long-term rate minus the short-term rate), and the SMB and HML factors of Fama and French (1993).[7] For reasons of tractability, we considered the effect of the addition of each factor by itself to the DDM in the same way that we considered the effect of the addition of turnover to the NVM. As one might expect, the addition of each factor helps explain the price-to-dividend ratio as compared with Shiller's DDM framework. The addition of our NV measure, however, results in a material outperformance of all the others. For example, the Shiller DDM's mean squared error (MSE) is 150.06 whereas the NVM's MSE is 101.09. The GDP model and the term-spread model have MSEs of 117.58 and 124.74, respectively. For the Fama–French factors (with data beginning in 1926), we found that the Shiller model yields an MSE of 164.16 whereas the NVM shows a significant improvement, yielding an MSE of 107.34. The SMB's MSE is 142.04 and the HML's MSE is 140.78, which represent only marginal improvements over the Shiller model.[8]

Data

Although others have used higher-frequency data, in our study we used low-frequency annual data for the following reasons: (1) conformity to the spirit of Shiller (1981), (2) avoidance of being stymied by seasonality in dividends, (3) availability of pre-1926 data, (4) lack of need for higher-frequency data to obtain statistically and economically meaningful results, and (5) lack of need for potential adjustment of the direct use of turnover. With higher-frequency data, our turnover function might need to be modified. As Pástor and Stambaugh (2003) suggested, volume can be high when liquidity is low; they noted the record high volume of 19 October 1987 as an example. An investor's trade volume might be high because of portfolio rebalancing during and after a sharp price decline. A sudden perceived deterioration in the effectiveness of the market network might cause some investors to demand less overall market participation or a different exposure to liquidity; thus, not counting such volume in building an NVM might be desirable.

For share turnover, we used annual NYSE Fact Book data.[9] We used the NYSE index level, together with its total return index level, to obtain the NYSE yearly dividends. For 1900–1925, we used the NYSE data in Goetzmann, Ibbotson, and Peng (2001); for 1926–2008, we used the NYSE data provided to us by Charles Jones (as used in Jones 2002). We confirmed that these data match CRSP data. We did not add net data on new issues to the dividend series.

Figure 5. Time-Varying NV Function, 1910–2008



Notes: This figure plots the values of the time-varying NV function used in the construction of $P^{(\dagger,*)}$ across decades. It shows not only how the value of the NV function changes over time but also how the NV function itself changes over time. The network effect follows the pattern of strong, subsiding, and then strong again.

Application to Puzzles

We believe that a viable economic justification exists for a network value theory, which, as we have shown, explains price well. We can now test and discuss the relationship of NV with the equity premium puzzle, the inverse inflation puzzle, and the news puzzle.

With low-frequency aggregate data, we can see the effects of NV on asset prices according to economic intuition—effects that describe asset prices better than the alternative whereby $\theta_{\mathcal{N}} \equiv 0$.

Equity Premium Puzzle. Cochrane (2001) and many others have deemed the equity premium puzzle crucial. Fama and French (2002) found that stock returns come primarily from a large capital gain and not from dividend growth, which prompts two important and related questions: (1) Why did this large capital gain occur? and (2) What is responsible for the change in the equity premium? Fama and French viewed the high *ex post* equity premium as an outcome that is "unexpected" by investors and that results from a new, lower discount rate (i.e., lower expected future returns). Shiller (1992) suggested "fads"; Keynes (1936) and Akerlof and Shiller (2009) proposed "animal spirits." Although consumption-based capital asset pricing, production-based asset pricing, and other models have met with some success, the equity premium puzzle still proves a challenge. Under our NV framework, a possible alternative to those views is that (1) returns vary because NV is priced with the market multiple, which has been higher in recent decades because NV has been higher, and (2) the unexpected part of growth in the price of stocks has come from the increasing aggregate NV.

Fama and French (2002) focused on the equity risk premium and how it was influenced by the contribution of capital gains to total returns over 1950–2000. We can use the NVM to further study the contribution to the large, unexpected capital gains over a similar period (1950–2007). To decompose $P^{(\dagger,*)}$ into two components, we can use the NV decomposition described in Equation 9, which allows us to separate our price approximation into the two terms from Equations 7 and 8. During the initial computation of $P^{(\dagger,*)}$, however, we have three contributors: dividends (the truncated sum in Equation 7), the network (the truncated sum in Equation 8), and the terminal value contribution (the appropriately discounted terminal value). For our approximations of P^{**} and $P^{\dagger}$, we allocate the terminal value contribution to $P^{(\dagger,*)}$ proportionally between the dividend and network contributions to $P^{(\dagger,*)}$. The percentage of the terminal value contribution added to the dividend contribution is the ratio of the dividend contribution to the sum of the dividend and network contributions. Similarly, the percentage of the terminal value contribution added to the network contribution is the same ratio but with the numerator replaced by the network contribution. We use these "grossed up" versions as the approximations of P^{**} and $P^{\dagger}$. Without loss of generality, we refer to these approximations as simply P^{**} and $P^{\dagger}$.

Figure 6 shows the contributions by P^{**}/D and $P^{\dagger}/D$ to $P^{(\dagger,*)}/D$ over time under the fixed NV function assumption. (The corresponding figure for the time-varying NV function is not shown because it looks, with the exception of its later start date, very similar to Figure 6.) We can see that the contribution from P^{**}/D has varied around a relatively constant mean since the beginning of the 1930s. Note the large recent contribution to price from $P^{\dagger}/D$, which has grown fairly steadily for about 50 years. The arithmetic mean of the historical yearly capital gain return over this period is 9.12 percent. This unusually large number lies at the heart of the equity premium puzzle. The NVM uses $P^{(\dagger,*)}$ to approximate P. This approximation's arithmetic mean of the historical yearly capital gain return is 6.81 percent. Although this average is lower than the realized capital gain average return, it is still large relative to the usual measures of the risk-free rate. Two sources that contribute to this unexpectedly large return are (1) the priced dividend growth and (2) the priced network effect.

Although this partition of the capital gain return contribution to total return is not a disjoint one because the network effect does indeed involve the covariation between the dividend process and the network production process, we can show that this network piece materially contributes to the capital gain return.

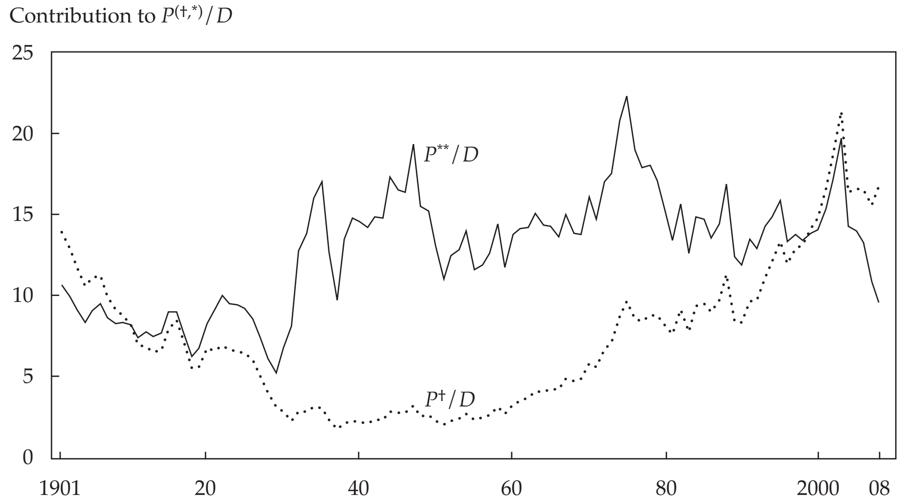
The following approximation displays the decomposition of the return into the sum of two returns:

$$\frac{\left(P_{t+1}-P_t\right)}{P_t} \approx \frac{\left[P_{t+1}^{(\dagger,*)}-P_t^{(\dagger,*)}\right]}{P_t^{(\dagger,*)}} = \frac{\left(P_{t+1}^{**}-P_t^{**}\right)}{P_t^{(\dagger,*)}} + \frac{\left(P_{t+1}^{\dagger}-P_t^{\dagger}\right)}{P_t^{(\dagger,*)}}. \tag{14}$$

We then define

$$r_t^{P^{**}} = \frac{\left(P_{t+1}^{**}-P_t^{**}\right)}{P_t^{(\dagger,*)}}$$

Figure 6. Contributions by Dividends and NV, 1901–2008



Notes: The *y*-axis is measured in index points and represents the contribution to the NVM's approximation of the actual price-to-dividend ratio. This figure clearly shows that the "functioning of the market" is not as large a part of our model's total price approximation from the 1930s through the 1970s as it is in the earlier and later periods.

and

$$r_t^{P^{\dagger}} = \frac{\left(P_{t+1}^{\dagger} - P_t^{\dagger}\right)}{P_t^{(\dagger,*)}}$$

so that

$$\frac{\left(P_{t+1} - P_t\right)}{P_t} \approx r_t^{P^{**}} + r_t^{P^{\dagger}}. \quad (15)$$

The linearity of the arithmetic mean allows us to decompose the average return of the realized capital gain approximately into an average return contribution from $r_t^{P^{**}}$ of 3.60 percent and an average return contribution from $r_t^{P^{\dagger}}$ of 3.21 percent. This 3.21 percent additional average return contribution from NV to the capital gain return helps explain the equity premium puzzle.

In recent years, research on state-contingent liquidity has attracted much attention. Neither Pástor and Stambaugh (2003) nor Liu (2006), however, addressed the question of why the priced liquidity factors they found should be so large. Amihud (2002, p. 33) stated that "liquidity is an elusive concept" and offered many possible definitions, all of which center on the concept of the "price to trade." Although this view is widely held in the academic and popular literature, liquidity factor payoffs seem far higher than what is required if liquidity is the cost of trading.[10] If investors really care so much about the current price to trade, why do they buy stocks? Does the varying state of the price to trade command such high premiums? Jones (2002) found that it does not.

Network value theory helps account for the observed large size of the liquidity premium. Liquidity is a part of the market network, whose value does vary, but the value of the market network grows over time and to a fairly high level. Previous liquidity studies have described the cross section well but have not provided a convincing new solution to the equity premium puzzle. Our argument—that increasing value in the market structure itself has produced these high observed returns, resulting in a market at a new, higher NV state—does help solve the equity premium puzzle. If liquidity is the cost to trade, it is a result of the network's existence and function. Liquidity thus defined is not NV because NV is more than the cost to trade. NV is the sum of the values of the network activities of the market participants who do trade, which is what has grown in value.

Finally, network value theory predicts that an improving market network helps illiquid stocks (e.g., small-cap stocks) the most and liquid stocks (e.g., large-cap stocks) the least. In this scenario, $P^{\dagger}$ may contribute to the size premium of Banz (1981). In Figure 6, we can see the high growth in $P^{\dagger}/D$ during well-known periods of high returns to small-cap stocks.

Inverse Inflation Puzzle. We have argued that NV is part of a stock's price. Because NV is a network and a network is a real asset, if NV does indeed possess state-contingent value, inflation should influence that value. The literature on stocks' inverse inflation relationship has assumed some form of valuation based on stocks' expected cash flows. NV has no cash flow except when investors sell stock holdings. Any unexpected inflation should negatively affect this part of a stock's value. Intuitively, investors would be expected to adjust their estimates of NV permanently downward on the basis of a long-term estimate of future inflation, consistent with studies that show a positive long-run Fisher (1930) effect on stock returns (see Anari and Kolari 2001). In the short term, however, inflation surprises should negatively affect NV—similar to a zero-coupon bond's negative reaction to inflation.

We can look at this short-term inverse inflation effect by relating changes in the $P^{\dagger}/D$ and P^{**}/D components to both the contemporaneous rate of change in the U.S. Consumer Price Index (CPI) and the previous year's change in the CPI (lagged), as shown in **Table 1**.[11] Specifically, for "contemporaneous" inflation, we used the rate of change in inflation between the CPI's two December levels over the previous year; for "forecast" inflation, we used the previous one-year lagged rate of change in the December levels. For both the CPI and the GDP, we used data from Shiller's website.[12] For our data, we could not separate expected inflation from unexpected inflation. Fama and Schwert (1977), however, found that both expected and unexpected inflation negatively affect stock prices.

Our hypothesis is that inflation adversely affects the network, which is more apparent when the network value is a higher proportion of total stock value. We can divide the time series of a stock's market value into sections for both the fixed NV function and the time-varying NV function, which are the two previously derived time series of prices shown in Figure 6. We can then run ordinary least-squares (OLS) regressions on each of the two time series, in which the dependent variable is either the part of the price-to-dividend ratio approximation from P^{**}/D, which corresponds to the dividend contribution, or the part from $P^{\dagger}/D$, which corresponds to the network contribution. These variables are then regressed against a constant and the CPI variable under consideration. Of the various "full period" regressions in Table 1, only one CPI coefficient based on P^{**}/D has a significant t-statistic, which is not surprising given the different states, or regimes.

Liu (2006) commented about various politico-economic events that affect liquidity and, therefore, asset prices. From the perspective of both market structure and the historical and current thinking about market structure, one may usefully divide the full universe of time series into three regimes: 1929 and before, the early regime; 1930–1971, the DDM regime; and 1972–2008, the NV regime. Klein (2001) and many others have discussed the rapid change in investors' perceptions, as well as those of concurrent economic thinkers, immediately after October 1929.[13] De Long and Shleifer (1991) argued that the large observed investment trust premiums of 1929 are evidence of a bubble. For the time series of both the fixed NV function and the time-varying NV function, $P^{\dagger}/D$ crossed below its full-universe mean after 1929 and remained there until the 1970s. The famous "beauty contest" analogy of Keynes (1936) was typical of the view that stock investing was not advisable for the general public or institutions (the U.S. Flow of Funds data show that individual investors directly owned more than 50 percent of equities until the early 1980s), a view that prevailed during both the early regime (pre-1929) and the DDM regime (1930–1971). From the 1930s through the 1950s, however, NV was at its lowest. During those years, dividend yield valuation methods became very important to investment practitioners and theorists. Graham and Dodd (1934) wrote the influential book *Security Analysis*, and Williams (1938) and Gordon and Shapiro (1956) developed the dividend discount model. When the dividend yield crossed the bond yield in 1958, many commentators believed (and it was widely reported at the time) that equities were priced too high.[14]

A very significant change in market structure began in December 1968, when the NYSE first allowed discounts for institutional (large block) trades. In 1970, institutional dollar volume exceeded 50 percent of all dollar volume on the NYSE. According to Jarrell (1984, p. 280), "In 1971 the SEC ordered that commissions be freely negotiated on any portion of an order above $500,000." This series of changes culminated in the so-called May Day event of 1 May 1975, when all NYSE commission rates became freely negotiable. At the time, these changes were considered watershed events, and they are still mentioned in the financial press. This deregulation enabled the dramatic decline in the costs of trading stocks (Jones 2002), and advances in technology and increased competition have allowed for even lower commission costs since then. The impact on trading costs of the NYSE's switch to "decimalization" (quoting of stocks in decimals as opposed to fractions) in 2000–2001 is less clear in the literature. Although we chose 1972 as the start of the NV regime, assigning 1976 as its first year made little difference in our results (Table 1).

Table 1. Regressions of $P^{\dagger}/D$ and P^{**}/D on Inflation

	Coeff.	*t*-Stat.	R^2	Coeff.	*t*-Stat.	R^2
Fixed NV function: Contemporaneous						
	1901–2008 (full period)			1901–1929 (early regime)		
P^{**}/D	11.84	1.80	0.030	–5.18	–1.65	0.091
$P^{\dagger}/D$	6.35	0.76	0.005	–3.53	–0.51	0.010
	1930–1971 (DDM regime)			1972–2008 (NV regime)		
P^{**}/D	20.88	3.01	0.184	31.57	2.34	0.135
$P^{\dagger}/D$	3.44	1.05	0.027	–59.09	–3.41	0.249
Time-varying NV function: Contemporaneous						
	1910–2008 (full period)			1910–1929 (early regime)		
P^{**}/D	13.72	2.09	0.043	–1.58	–0.35	0.007
$P^{\dagger}/D$	9.06	1.03	0.011	–1.72	–0.62	0.021
	1930–1971 (DDM regime)			1972–2008 (NV regime)		
P^{**}/D	19.84	2.99	0.182	33.48	2.37	0.138
$P^{\dagger}/D$	5.92	1.61	0.061	–55.83	–3.27	0.234
Fixed NV function: Forecast						
	1902–2008 (full period)			1902–1929 (early regime)		
P^{**}/D	6.61	0.99	0.009	–3.39	–1.10	0.044
$P^{\dagger}/D$	5.60	0.67	0.004	–4.11	–0.66	0.016
	1930–1971 (DDM regime)			1972–2008 (NV regime)		
P^{**}/D	10.09	1.34	0.043	16.28	1.14	0.036
$P^{\dagger}/D$	1.29	0.39	0.004	–68.00	–4.15	0.330
Time-varying NV function: Forecast						
	1910–2008 (full period)			1910–1929 (early regime)		
P^{**}/D	7.49	1.12	0.013	–1.47	–0.33	0.006
$P^{\dagger}/D$	8.13	0.09	0.009	–1.29	–0.46	0.012
	1930–1971 (DDM regime)			1972–2008 (NV regime)		
P^{**}/D	9.84	1.37	0.045	17.23	1.15	0.037
$P^{\dagger}/D$	3.72	0.99	0.024	–65.16	–4.05	0.319

Notes: In general, the impact of a stock's inverse inflation relationship comes from $P^{\dagger}/D$, consistent with the NV model's prediction. This table shows OLS regressions of $P^{\dagger}/D$ and P^{**}/D decompositions for time t on inflation, which is the percentage change in the CPI (December to December) for time t (contemporaneous) and time $t-1$ (forecast). "Fixed NV function" means that the discount rate, δ, and the NV function, $\theta_{\mathcal{N}}$, are fixed. "Time-varying NV function" means that both the discount rate and the NV function vary.

Interestingly, in both the fixed and the time-varying contemporaneous regressions of P^{**}/D, we found that the CPI coefficient is significantly different from zero for both the DDM regime and the NV regime (1930 and beyond). For all the regressions of P^{**}/D, all the CPI coefficients are positive except for those of the early period. Also, for the contemporaneous full-period regressions under both the fixed and the time-varying NV functions, the t-statistics associated with the CPI variables are at or near levels of significance. After decomposing the price-to-dividend ratio into $P^{\dagger}/D$ and P^{**}/D, we can see that P^{**}/D increases with inflation—that is, the Fisher relationship holds.

Intriguingly, Kaul (1987, 1990) and Lee (2009) found that stock values in the prewar period did not have the inverse inflation relationship that researchers found them to have in the postwar period. Modigliani and Cohn (1979) used a "money illusion" theory[15] to explain stocks' inverse inflation relationship, arguing that "investors capitalize equity earnings at a rate that parallels the minimal interest rate on bonds, rather than the economically real rate" (p. 24)—often referred to as the Fed model fallacy. The argument that investors make a systematic error in pricing stocks by incorrectly relying on a simple nominal pricing model has two problems not emphasized in other studies. First, Schwert (2003) showed that after discovery, anomalies tend to weaken or disappear. That the inverse inflation relationship has gotten stronger since 1979, not weaker[16]—in spite of the fact that it has attracted intense academic interest—bodes poorly for a behavioral explanation. Second, that the money illusion literature contains no studies of direct investor behavior (e.g., equity fund flows)[17] to buttress that argument seems curious. Our competing explanation is the very simple idea that a network is a real asset, and real assets are adversely affected by shocks to inflation. In the earlier periods in Table 1, investors may not have been aware of the importance of NV; indeed, NV was not especially important for much of that time. If investors were ever fooled, that was probably during the DDM regime (1930–1971), when the CPI coefficient for the $P^{\dagger}/D$ contemporaneous regression had the wrong sign, although it was relatively small. After 1971, under both the fixed and the time-varying NV functions, the contemporaneous CPI coefficients relative to $P^{\dagger}/D$ have the predicted sign and are both large in magnitude and highly significant—all evidence that investors are aware of NV's existence and correctly price inflation shocks. Moreover, our explanation, which accounts for the direct adverse effect of inflation on NV, does not rely on an indirect adverse effect (Roll and Geske 1983) or on the idea of inflation as a proxy (Fama 1981). The strength of our explanation and its consistency with economic insight suggest a useful solution to the inverse inflation puzzle.

Finally, Table 1 also shows the results for the forecast regressions. A lagged change in the CPI with the coarse time-series data that we used would not be expected to produce a well-fitted forecast, and we found that the CPI coefficients are almost all smaller in magnitude and have the same sign as under the contemporaneous regressions. As expected, these results are generally weaker than those for the contemporaneous regressions, except during the NV regime, when the CPI coefficients and *t*-statistics associated with the $P^{\dagger}/D$ regressions are of slightly higher magnitude—that is, a lagged change in the CPI does forecast a decrease in this part of a stock's value.

News Puzzle. The news puzzle is not that stocks move irrationally because of the first term in Equation 4. According to Cochrane (1991, p. 480), "When there is fundamental news, the markets react by about the right amount. The puzzle is that prices also move when there is no obvious news." Cutler, Poterba, and Summers (1989) discussed the apparent puzzle of large stock price movements and the lack of any corresponding news that should affect stock price fundamentals: "Our results suggest the difficulty of explaining as much as half of the variance in aggregate stock prices on the basis of publicly available news bearing on fundamental values" (p. 9). Jacklin, Kleidon, and Pfleiderer (1992) and Grossman (1988) built information models in their attempts to supply a theory to account for the large price decline of 19 October 1987. In general, researchers have not provided a fundamental rationale for the crash; explanations have generally focused on the market's functioning, consistent with our thesis.

Finance and investment texts often discuss the "real" and the "financial" sectors, with the financial sector pricing financial claims without friction and not producing a wedge in prices. As we have argued, the *network value* of stocks is priced—not just a wedge but also an expansion—and thus new information about the performance of the market in its role as a provider of this network is important. Viewed in this way, the market making its own news is not a puzzle. What happens to the market is news. A market malfunction, such as the crash of 19 October 1987 or the credit crisis of late 2008, should cause the value of stocks to fall.

Conclusion

We have argued for the addition of a term to valuation and asset pricing models that incorporates the functioning of the market exchange. This idea is the network value theory. We used network structure arguments to assign a price to the network and then used turnover to build a version of NV. When we allow for the existence of NV, we are able to see immediately that it can be helpful in concisely addressing well-known asset pricing puzzles. Network value theory offers convincing ways to solve the equity premium, inverse inflation, and market-making-its-own-news puzzles.

Asset pricing theory has relied heavily on the assumption of complete markets, in which everyone has complete information and can trade instantly with anyone, anywhere, at no cost. But that is a simplifying assumption. Amplifying O'Hara (2003), the "as if" of Friedman (1953, p. 40) has become today's "as is." If everyone "knew" everything, there would be no need for—or value of—the internet; and if product markets were perfectly liquid, there would be no point in having inventory. These activities have costs but also contribute value. Similarly, if we assume that everyone does not know everything about expected payoffs for asset prices and that the facilitation of trade is not free—that investors instead rely on a network to help determine prices (information production) and to facilitate and finance trading (liquidity) and that this network has value—then we have our network value.

Interestingly, the portion of a stock's value contributed by NV is consistent with the body of research that demonstrates a premium value for publicly traded stocks vis-à-vis nonpublicly traded stocks (see, e.g., Silber 1991). Longstaff (2009, p. 1122) provided a theoretical liquidity model whereby "heterogeneity in patience" between investors can have large asset pricing implications. This type of theoretical work can be considered "on the demand side" and helps answer the question, What are investors willing to pay for convenience? Network value theory can be considered "on the supply side" and helps answer the question, How do liquidity and information suppliers structure all their activities, and what is the value of that? Our study addresses the latter.

Some might object that our NV function mistakenly ascribes value to the "noise traders." As we have shown in our discussion of inflation, however, the market prices NV, which has a "real" value. Perhaps some of the volume is "irrational" and NV is too large. The current asset pricing theory alternative, however, is $\theta_{\mathcal{N}} \equiv 0$, which produces the puzzles. Assuming a nontrivial $\theta_{\mathcal{N}}$ helps solve the puzzles. In addition, the assumption that investors are rational is the usual starting point in economic model building. Further, the news anomaly research has found large stock price swings (regularly accounting for half the variance) for reasons other than news pertaining to stock earnings. Are investors really so irrational that they needlessly incur trading costs, or are they impounding news pertaining to NV? French and Roll (1986) showed that stock return variation is higher when the market is open, which is exactly consistent with our NV model.

A myriad of related issues await further research: NV and the closed-end-fund puzzle; the NV expected contribution to the returns of other markets, especially emerging markets (although consistent volume data, even within the same market, are not easily available); other ways to measure NV; and disentangling NV from other factors, such as size. Also, our approach to modeling the various network states could be considered for other networks. Although we have shown that growth in network value is quite steady and is a material part of the increase in a stock's value, that this improvement in financial intermediation might provide a similar boost to prices in the next 50 years is difficult to imagine. Nevertheless, that a financial production function exists (Snigaroff 2000)—that real and measurable value is built by the network of market practitioners—certainly seems less bleak than the nihilistic assumption that their efforts contribute nothing.

We are grateful for helpful discussions with Allan Timmermann and Bruce Lehman, both at the University of California, San Diego, and with Mike Munson at Denali Advisors.

This article qualifies for 1 CE credit.

Appendix A. From the DDM to the NVM

This appendix shows how our NVM differs from the DDM and formally defines the NV function, θ.

Dividend Discount Model

Beginning with

$$R_t = \frac{\left(P_{t+1} - P_t + D_t\right)}{P_t}, \tag{A1}$$

we take expectations, assume that $\mathrm{E}(R_t \mid \mathcal{I}_t) = r, \forall t$, and obtain

$$P_t = \frac{1}{(1+r)} \mathrm{E}\left(P_{t+1} + D_t \mid \mathcal{I}_t\right). \tag{A2}$$

Iterating this process, using the tower property of conditional expectation, and assuming that the growth rate of the numerator is less than the discount rate, r, we obtain the dividend discount model

$$P_t = \sum_{k=0}^{\infty} \frac{\mathrm{E}\left(D_{t+k} \mid \mathcal{I}_t\right)}{(1+r)^{k+1}}, \tag{A3}$$

which leads to an expression for the price-to-dividend ratio by dividing by D_{t-1}:

$$\frac{P_t}{D_{t-1}} = \sum_{k=0}^{\infty} \frac{\mathrm{E}\left[\left(D_{t+k} / D_{t-1}\right) \mid \mathcal{I}_t\right]}{(1+r)^{k+1}}. \tag{A4}$$

Note on the Functional Form

The specific bounded function that we use to accomplish this idea of increasing followed by decreasing returns to scale in the network formation is the cumulative distribution function associated with the normal distribution, a readily recognized function. Although we believe that a linear function is an incorrect economic specification, we tried that approach and obtained somewhat weaker results.

(The real value of our argument is in the addition of a term for the value of stocks; the choice of function is not the most significant part of our argument.) Thus, we define the NV function, $\theta_{\mathcal{N}}$, through the cumulative distribution function, Φ, of a normally distributed random variable as

$$\begin{aligned}\theta_{\mathcal{N}}(x) &= \alpha\left[\Phi_{(\mu,\sigma^2)}(x) - \Phi_{(\mu,\sigma^2)}(0)\right] \\ &= \alpha\left[\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{x} e^{\frac{-(y-\mu)^2}{2\sigma^2}}\,dy - \Phi_{(\mu,\sigma^2)}(0)\right],\end{aligned} \quad \text{(A5)}$$

where the subscript $\mathcal{N}$ on the NV function θ indicates the NV function's dependence on the underlying time series used to measure the network's production. We also estimate the parameters μ and σ by the usual point estimates

$$\hat{\mu} = \frac{1}{n}\sum_{k=1}^{n}\mathcal{N}_k$$

and

$$\hat{\sigma} = \sqrt{\frac{1}{n-1}\sum_{k=1}^{n}\left(\mathcal{N}_k - \hat{\mu}\right)^2},$$

where n represents the number of data points in the sample. The parameter α, which represents the scaling of the function in the fit, is given by the estimate provided by an application of a nonlinear least-squares regression that estimates the pair (δ, α) in the return equation given by Equation A6.

We incorporate the NV function into a DDM framework by expressing the total return as a constant minus our function of network production. To stay partly in the DDM framework, we must divide our function by the current price. This step also allows us to keep our network component separate and to prevent it from being completely algebraically absorbed into the discount rate. This formulation, however, leads to the problem of an unbalanced regression model because the returns are being modeled by a constant minus the ratio of a bounded function to a growing price series. Thus, our additional term is decaying and contains a time dependence not inherent in the underlying return series. We thus multiply our additional component by the dividend series. The result is that our function of the network proxy is multiplied by the dividend yield, which creates a balanced model and brings us closer to stationarity as a whole. We thus arrive at Equation A6, which gives the future return process:

$$\begin{aligned}\frac{\left(P_{t+1} - P_t + D_t\right)}{P_t} &= R_t = \delta \\ &- \frac{(1+\delta)\left[\theta_{\mathcal{N}}(\mathcal{N}_t)\right](D_{t-1})}{P_t} + \epsilon_{t+1},\end{aligned} \quad \text{(A6)}$$

where ϵ is an i.i.d. (independent and identically distributed) process with respect to time that satisfies $E(\epsilon_{t+1} \mid \mathcal{I}_t) = 0$ for each time t. Taking expectations, we obtain

$$\begin{aligned}P_t &= \frac{1}{(1+\delta)}E\left(P_{t+1} + D_t \mid \mathcal{I}_t\right) \\ &+ E\left[\theta_{\mathcal{N}}(\mathcal{N}_t)(D_{t-1}) \mid \mathcal{I}_t\right].\end{aligned} \quad \text{(A7)}$$

Proceeding as before with the DDM and assuming that the sums are finite, we obtain

$$\begin{aligned}P_t &= \sum_{k=0}^{\infty}\frac{E\left(D_{t+k} \mid \mathcal{I}_t\right)}{(1+\delta)^{k+1}} \\ &+ \sum_{k=0}^{\infty}\frac{E\left[\theta_{\mathcal{N}}(\mathcal{N}_{t+k})(D_{t-1+k}) \mid \mathcal{I}_t\right]}{(1+\delta)^k}.\end{aligned} \quad \text{(A8)}$$

Then, dividing by D_{t-1}, we obtain an expression for the price-to-dividend ratio:

$$\begin{aligned}\frac{P_t}{D_{t-1}} &= \sum_{k=0}^{\infty}\frac{E\left[\left(D_{t+k}/D_{t-1}\right) \mid \mathcal{I}_t\right]}{(1+\delta)^{k+1}} \\ &+ \sum_{k=0}^{\infty}\frac{E\left[\theta_{\mathcal{N}}(\mathcal{N}_{t+k})(D_{t-1+k}/D_{t-1}) \mid \mathcal{I}_t\right]}{(1+\delta)^k}.\end{aligned} \quad \text{(A9)}$$

Appendix B. Technical Information on the Superior Fit of the NV Model

This appendix provides technical information on the superior fit of our NV model.

Statistical Fit

Table B1 provides summary statistics based on the data from the NV and Shiller approximations of P/D in both the fixed NV function and the time-varying NV function cases. The table confirms the graphical depictions in Figures 2, 3, and 4. We also use three common measures of closeness in the density space: the Kullback–Leibler distance and the L^1 and L^2 norms (their formal definitions appear later in this appendix). Our NV model displays a better statistical fit than the Shiller model in all the metrics except one (the ratio of standard deviations).

With respect to the ratio of standard deviations, the literature at the time of Shiller (1981) unambiguously favored ratios near to or greater than 1. Today, the literature suggests that this condition is not so important. One of Shiller's arguments concerning volatility is based on the ratio of variances: Assuming that a detrending step produces stationary processes and that the proposed model for

Table B1. Summary Statistics for the NV and Shiller Approximations of *P/D*

Comparison	Fixed Shiller P^*/D	Fixed NV $P^{(\dagger,*)}/D$	Time Varying Shiller P^*/D	Time Varying NV $P^{(\dagger,*)}/D$
Mean squared error:	150.06	101.09	273.08	123.51
$\frac{1}{T}\left\lVert \widehat{P}/D - P/D \right\rVert^2_{l^2(\{1,2,\ldots,n\})} = \frac{1}{n}\sum_{k=1}^{n}\left(\frac{\widehat{P}_k}{D_{k-1}} - \frac{P_k}{D_{k-1}}\right)^2$				
Standard deviation ratio: $\frac{\sigma_{\widehat{P}/D}}{\sigma_{P/D}}$	0.82	0.54	0.76	0.59
Correlation of P/D and $\widehat{P}/D$	0.30	0.70	–0.05	0.71
Density approximations				
Kullback–Leibler distance $(P/D, \widehat{P}/D)$	0.35	0.19	0.47	0.31
L^1 distance	0.70	0.43	0.83	0.55
L^2 distance	0.13	0.08	0.14	0.10

Notes: This table presents the statistics for goodness of fit. It compares the NV and Shiller approximations of the actual price-to-dividend ratio in both the fixed NV function and the time-varying NV function cases.

prices is correct, Jensen's inequality implies that the var(P^*/D) ≥ var(P/D). We point out, however, that this argument breaks down if the detrending step does not result in a stationary process. If the stationarity assumption does not hold, then we may conclude only that $\text{var}\left(P_t^*/D_{t-1}\right) \geq \text{var}\left(P_t/D_{t-1}\right)$ for each fixed time t and we may not draw the previous conclusion regarding variation over time from this spatial variance inequality, which may hold only for each fixed time t. Therefore, the fact that this inequality across times is violated when using the sample data—in both Shiller's model and the NV model (see Table B1)—is not in itself a contradiction of the efficient market model. An estimator that displays more volatility over time does not imply that it is superior to one that displays less volatility. As we can see in Figures 2 and 3 and in Table B1, greater variation over time does not imply that a model will have a closer fit, even allowing for different measures of closeness.

Forecast Encompassing

We also ran two regressions involving the price-to-dividend ratio squared-error processes of both the Shiller model and the NV model to gauge the forecasting abilities of each model. We first regressed the Shiller model's squared error against a constant and $P^{(\dagger,*)}/D$. We found that the t-statistic (5.12) of the coefficient for $P^{(\dagger,*)}/D$ is significant, and thus the fixed NV model helps explain the Shiller model's error process. We also regressed the NV model's squared-error process against a constant and the Shiller approximation of P/D, or P^*/D. We found that the t-statistic (1.28) of the coefficient for P^*/D is not significant and that we thus cannot conclude that the Shiller model explains the NV model's error terms. Therefore, the NV model is said to *forecast encompass* the Shiller model, which demonstrates the NV model's superior explanatory power. For the time-varying function, the forecast-encompassing results for the NV model compared with those for the Shiller model yield t-statistics of 1.22 and –1.39 in the same order as previously described. Because both t-statistics are not significant, we are unable to say that either model forecast encompasses the other.

Density Measures of Closeness

The Kullback–Leibler distance is defined by

$$\text{E}_f\left\{\ln\left[\frac{f(X)}{g(X)}\right]\right\} = \int_{-\infty}^{\infty}\ln\left[\frac{f(x)}{g(x)}\right]f(x)\,dx. \quad \text{(B1)}$$

The L^1 and L^2 norms are given by the following formulas:

$$\begin{aligned} \lVert f-g\rVert_{L^1} &= \int_{-\infty}^{\infty}\lvert f-g\rvert\,dx; \\ \lVert f-g\rVert^2_{L^2} &= \int_{-\infty}^{\infty}(f-g)^2\,dx. \end{aligned} \quad \text{(B2)}$$

Notes

1. Shiller (1981) used the term *efficient market model*, which we use here.
2. As we discuss later in the article, we used a function of *turnover* to build our network value function. At times, however, we use the term *volume* to refer to the level of trading on the exchange. Both terms refer to the number of shares, not dollars.
3. The intuition for confidence is simple; similar to the value of the internet increasing marginally with traffic as the number and frequency of users increase with users' expectations of traffic, the market's network value increases with more participants and a greater volume of trading.
4. See Appendix A for the details of the calculation.
5. Graphical examples of this function are given in Figure 5.
6. That is, share turnover from an econometric perspective; share volume standardized via division by shares outstanding is preferred to volume alone. Turnover is a composite of volume and shares outstanding, however, and Pontiff and Woodgate (2008) demonstrated that the number of shares outstanding for stocks describes security returns in the cross section. We also built an NV function by using a volume series V in place of $\mathcal{N}$ with the following detrending procedure: We first regressed $\log(V_t)$ against $a + bt$ and used e^b as the long-term growth rate. The detrended series is then given by $v_t = (V_t)e^{b(T-t)}$, which is equivalent to growing the time t volume at the long-term growth rate until terminal time T. We obtained materially stronger results for this NV function than for the NV function with turnover in each of our puzzles (discussed later in the article). The use of detrended series has issues of its own, however, and we show only results for the version of NV with turnover. Bruce Lehman and Allan Timmermann provided helpful comments on the detrending issue.
7. For the annual nominal GDP and the long-term rate (10 years), we used the data from R.J. Shiller's website (www.econ.yale.edu/~shiller/data.htm). For the short-term rate, we used the "Bills - nominal - rates" series from Jeremy Siegel's website (www.jeremysiegel.com). The Fama–French factor return data consist of the historical benchmark returns from Kenneth R. French's website (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french)—in particular, the annual series. According to the "Description of Fama/French Benchmark Factors" section of the website, "SMB (Small Minus Big) is the average return on three small portfolios minus the average return on three big portfolios," and "HML (High Minus Low) is the average return on two value portfolios minus the average return on two growth portfolios."
8. Under other methods of comparison (see Table B1 in Appendix B), the NVM generally outperforms each of the other variables.
9. We used data from the "NYSE Historical Statistics" tables at www.nyxdata.com. In the NYSE Fact Book, yearly share turnover "is calculated by multiplying the year-to-date average daily volume by the number of trading days in the current year and dividing by the average of total shares outstanding at the end of the previous year and total shares outstanding at the end of the given month [for December, the year-end shares outstanding]."
10. Lo, Mamaysky, and Wang (2004) seemed to offer a reason for such large premiums in proposing a power law function with respect to fixed trading costs. Arguing that "small fixed costs can have a significant impact on asset prices," they suggested that "most dynamic equilibrium models will show that it is quite rational and efficient for trading volume to be infinite when the information flow to the market is continuous" (p. 1056) and demonstrated with their model that even moderate costs lead to "large 'no-trade' regions" (p. 1054).
11. For an excellent review of the various techniques applicable to the study of stocks' inverse inflation relationship, see Anari and Kolari (2001).
12. For inflation, we used data from the annual CPI-U (Consumer Price Index for All Urban Consumers) as obtained from Shiller's website (www.econ.yale.edu/~shiller/data.htm); for years prior to 1913, the annual CPI-U data are "spliced to the CPI Warren and Pearson's price index." Shiller's website has two sets of CPI data; we used the rate of change in the monthly December index levels from the "Irrational Exuberance" worksheet. Shiller's annual set of data is also available but uses a January end date to conform to his earlier work.
13. The infamous quote attributed to Irving Fisher—"Stock prices have reached what looks like a permanently high plateau"—may be apocryphal (see Klein 2001). Klein also described Fisher's public concern about deflation, which began in January 1930.
14. We have read numerous stories in the *New York Times* from 1958 to 1962 and beyond that discuss this event, which was viewed as a novelty. At the time, however, some commentators asserted that the same thing had also occurred in 1929.
15. This theory is one of the leading contenders, among many, for solving the inverse inflation puzzle. For one of the more recent and complete reviews of this literature, see Lee (2009).
16. Using our annual data and model, we found the effect strongest after 1979.
17. Having evaluated investment management firms over the course of several hundred visits, we can recall only one asset manager who admitted to using the Fed model in valuation work (the admission was vivid because it was widely viewed as naive—a view the firm acknowledged without prompting). Still, the manager remained at least 95 percent invested in stocks, as is typical of all institutional investment managers.

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